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NAVAL POSTGRADUATE SCHOOL Monterey, California



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A STUDY OF MULTI-ECHELON
AND MULTI-LOCATION INVENTORY SYSTEM

by

Turgut Büyükkarhan

September 1980

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The results from the mathematical model were used as input values for the simulation and measures of effectiveness were compared. An alternative procedure was proposed and simulated; and the results of the three products were compared.

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A Study of Multi-Echelon

And Multi-Location Inventory System

by

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Submitted in partial fulfillment of the requirements for the degree of

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ABSTRACT

The reorder point and reorder quantity for a multiechelon inventory system consisting of two levels were determined through the use of a mathematical model and a computer simulation was used to verify the results.

The main echelon supported two lower echelon stock points and reordered using a continuous review inventory policy. The two lower echelon stock points operated under periodic review policies.

The measure of effectiveness used was to minimize total system costs subject to a constraint on the maximum number of back orders per year.

The results from the mathematical model were used as input values for the simulation and measures of effectiveness were compared. An alternative procedure was proposed and simulated; and the results of the three products were compared.

TABLE OF CONTENTS

I.	INT	RODUCTION	8
II.	A PI	ROBABILISTIC MATHEMATICAL MODEL FOR MULTI- ELON INVENTORY SYSTEM1	0
	A. B. C.	DESCRIPTION OF MODEL	0
		1. Main System	1
	D.	ANALYTICAL SOLUTION TO THE MODEL15	5
		 Main System	5 9
		a. Ordering and Reviewing Costs	9
		3. Objective Function and Constraints22 4. Optimum Values of Operating Variables23	2
	Ε.	EXAMPLE PROBLEM26	5
		1. Additional Assumptions26 2. Input Values27	5
		a. Main System28 b. System One28 c. System Two28	3
		3. Solution Procedure and Results29)
III.	COM	PUTER SIMULATION FOR MULTI-ECHELON, MULTI- ATION SINGLE ITEM INVENTORY SYSTEMS36	;
	Α.	DESCRIPTION OF THE SIMULATION MODEL36	;
		1. Main System36 2. System One and System Two38	

	B. C.	HOW RUNS	TO U	JSE TH	HE P	ROG	RAM-								- 38 - 39
		1.	Star	rting ults o	Con	dit	ions								- 39 - 40
			a. b. c.	Main Syste Syste	em Ω	ne-							_ ~		- 40
				(1)	Tot	al	Resu	ılts	5						- 41
					(a) (b) (c)	S	vste	m (ne-						- 42
				(2)	Tot	al	Resu	ilts	3						- 43
		3.	Comp	pariso	on o	f B	oth	Sta	arti	ng C	ond:	tio	ns		- 43
IV.	COM RES	PARIS ULTS-	SON (OF ANA	LYT	'ICA	L AN	ם מו	COMP	UTER	SI	IULA'	TION	N	_ 44
v.	AN	ALTE	RNAT	IVE S	OLUT	ION									- 49
	A. B.	COM	ואיינזכ	rion (R Simu	IT.AT	'ION	RES	ULT	rs F	OR E	ARL	WA.	RNII	NG	
		1. 2. 3.	Syst	n Syst tem Or tem Tr	ne R	26511	lts-								- 51
	c.	RESI	TTTS	SON OI WITH	ANA	LYT	TCAT	. Al	ND F	IRST	SI	/ULA	OIT	N	- 51
VI.	CON	CLUS	ION-												- 53
APPENI	XIC	A - 1	Flow	chart:	s of	Fi	rst	Sir	nula	tion	Mo	del-			- 56
APPENI															
APPENI	DIX	C - :	Simu	latio	n Pr	ogr	am;	Ve	rsat	ec F	lot	cer-			_ 80
APPENI	DIX			lation			am;	No	Ver	sate	c P	lott	er 		-101
ADDENI	ntv	F	Farl	u War	nino	, Ci	mu l a	a + i /	an D	roar	am-				-120

APPENDIX	F -	Versatec	Output	of the	Program	in Ap	ppendix	C-	138
APPENDIX	G -	TI-59 Ca. Solution							141
APPENDIX	н -	Variable	Definit	ions fo	or Simula	ation	Program	ıs-	146
BIBLIOGRA	/PHY-								148
INITIAL D	ISTE	RIBUTION	LIST						149

I. INTRODUCTION

Despite the fact that all military supply systems and many large civilian corporations have multi-echelon and multi-location inventory systems, few multi-echelon inventory models are currently being used. The policies that have been used are the single echelon and single location inventory policies that attempt to minimize the total variable cost of a single location ignoring total system cost. The difficulties of optimizing a multi-echelon and multi-location inventory system are most likely due to the complexity of demand and the interdependency among the units in different echelons.

A mathematical model for a multi-echelon and multilocation inventory system is developed in Chapter II. The
objective of this model is to minimize the total annual
variable costs subject to a maximum allowable number of
back orders per year for the entire system. This model
consists of one first echelon location operating under a
continuous review policy and two second echelon systems
which operate under periodic review policies. An example
is presented to obtain numerical results.

In Chapter III, a computer model is developed to simulate the multi-echelon, multi-location system. This simulation model was used to check the results obtained from

the mathematical model and to demonstrate the interaction between echelons. Graphical plots are generated which show the inventory position of each entity in the model.

In Chapter IV, comparisons are made of the simulation results and the results obtained from the mathematical model. The comparisons suggest some modifications to the operating policy.

The system was simulated again using the modified operating policy and substantial improvements in the effectiveness of the multi-echelon system were observed. These results are described in Chapter V.

Chapter VI summarizes the results of the research and concludes with some suggestions for additional research.

II. A PROBABILISTIC MATHEMATICAL MODEL FOR MULTI-ECHELON INVENTORY SYSTEM

A. DESCRIPTION OF MODEL

In this model there are three systems. One of these systems represents the highest echelon and is called the Main System. The other two, at separate locations, are the lower echelon and are called System One and System Two, respectively. Each system carries its own inventory, and receives random demands. System One and System Two are dependent upon the Main System but independent of each other. System One and System Two can only be resupplied by the Main System. In other words, they cannot order from any other suppliers. The Main System replenishes stocks by placing orders to external suppliers.

B. OBJECTIVE OF MODEL

The objective of this model is to minimize the total expected annual variable costs of three systems subject to a specified expected number of total back orders per year. In fact, this is the same as minimization of total yearly variable costs subject to a specified minimum level of customer satisfaction.

Based on these objectives, decision variables for each system will be calculated to achieve optimum levels for the entire multi-echelon supply system.

C. ASSUMPTIONS OF THE PROBABILISTIC MATHEMATICAL MODEL

1. Main System

It is assumed that the Main System under consideration consists of a single installation which utilizes transaction reporting. This system is governed by a (Q, r) type policy with back orders.

The assumptions in addition to the continuous review assumption are:

- a. The cost of operating the information processing system is independent of Q (reorder quantity) and r (reorder point).
- b. The unit cost C of the item is a constant independent of Q.
- c. The back-order cost is constant (Π) , per unit back ordered regardless of the length of time the back-orders exist.
- d. There is never more than a single order outstanding. This assumption implies that when the reorder point is reached, there are no orders outstanding; therefore, the inventory position is equal to the net inventory. Thus, the reorder point will be the same regardless of whether it is based on the inventory position or net inventory.
- e. Procurement lead times are independent and identically distributed (i.i.d) random variables with a gamma distribution.

- f. All variables are treated as continuous.
- g. The demands are Poisson distributed with the mean number of demands per year a constant λ_m .
- h. The reorder point, r, which is based on the inventory position is positive.

With a back orders constraint, it is infeasible to wait until back orders exist before placing an order.

Because of this and assumption (d) there will be no back orders outstanding at the reorder point. As was discussed in assumption (d), at the reorder point, the inventory position is equal to the on-hand inventory.

For this model, any one of the three inventory levels on hand, net, or inventory position can be used to define the reorder point; and the reorder point has the same value for any one of them. It should also be noted that to use the on-hand level it must be assumed that after an order arrives, it is sufficient to fill all back orders and raise the on-hand inventory level above the reorder point. If this ever failed to happen, the reorder point would never be reached again and the system would continue to accumulate back orders indefinitely.

When the reorder point is thought of in terms of the inventory position of the system, then assumption (d) guarantees that the on-hand inventory will always exceed the reorder point when an order arrives; otherwise, it would not be possible to have only a single order outstanding.

2. System One

This system, which is at the lower echelon, is governed by a periodic review policy. The operating doctrine is the most widely used type of periodic review which is the order up to R policy. All demands which occur when the system is out of stock are back ordered.

For this periodic review system the time between reviews will be denoted by T, and at each review time a sufficient quantity is ordered to bring the inventory position of the system up to a level, R, regardless of the amount of on-hand inventory. This policy dictates that at review times even if the inventory position is R-1, only one item must be ordered to bring the inventory position up to R, ignoring the high cost of placing the order. Despite its appearance of being illogical, this assumption simplifies the formulation of the system and is very unlikely to occur on high demand items. When an item experiences zero demand in a review period, there is no need for order because the inventory position is already at R.

The other assumptions are:

- a. The cost, J, of making a review is independent of the variables R and T.
- b. The unit cost, C, of the item is constant and independent of the quantity ordered.

- c. Back orders are incurred only in very small quantities. This implies that when an order arrives, it is almost always sufficient to meet any outstanding back orders.
- d. The backorder cost is constant, \mathbb{N} , per unit back ordered regardless of the length of time the back order exists.
- e. Procurement lead times are i.i.d random variables with a gamma distribution.
- f. Orders are received in the same sequence in which they were placed. It should be noted that for (Q, r) models, the two assumptions that orders were received in the sequence placed and that lead times are i.i.d random variables could not both hold rigorously, since there exists a positive probability that two successive orders could be separated by an arbitrarily short time interval. In this model, orders can never be more closely spaced than by an interval of length T. If T is large enough, it is possible, provided that there is a sufficiently small range of variation in the lead time, that both assumptions hold simultaneously.
- g. The demands are Poisson distributed with mean, $\boldsymbol{\lambda}.$
 - h. All variables are treated as continuous.

3. System Two

This system is identical to, but independent of, System One.

D. ANALYTICAL SOLUTION TO THE MODEL

As was mentioned previously, the objective of the model is to minimize total system costs subject to a constraint on the number of back orders per year.

The determination of various annual costs of each system may be made independently and placed into a common cost formula. Instead of doing this independently, it is better to find a main system cost expression, System One cost expression and System Two cost expression, and then to add them to each other to determine the total cost formulation and then use this formulation as the objective function of total system. The systems are actually tied together through the constraint on the number of back orders.

For the following notation, the subscript m will indicate that the variables being subscripted belong to the Main Sytem where 1 and 2 indicate System One and System Two, respectively.

The costs that are of interest in this model for each system are the cost of placing an order, the cost of carrying inventory and back order costs.

1. Main System

As was discussed previously, the Main System has a transactions reporting policy or (Q, r) model.

In a continuous review system, a period is defined as the length of time between the receipt of two successive procurements. This time period is a random variable.

Because the procurement lead times are random variables, the number of demands for a fixed time are random variables; and the number of items demanded per demand are also random variables. The review period is also a random variable.

In the following material, we itemize the costs.

a. Procurement Cost

 $\lambda_{\rm m}$ = Expected number of demands per year

 $Q_m = Order quantity$

 $\frac{\lambda_{m}}{Q_{m}}$ = Average number of procurements per year

 A_{m} = Procurement cost per cycle

 $\frac{\lambda_{m}}{Q_{m}} A_{m} = Procurement cost per year$

b. Inventory Carrying Costs

Because of randomness, it is possible to accumulate a large number of back orders at the end of a cycle.

To prevent this from occurring, it is advisable to provide a safety stock to buffer the system from excessive numbers

of back orders. The safety stock is the expected amount of stock on hand when an order arrives. The actual amount of stock on hand when a shipment arrives is clearly random.

S_m = Mean value of on-hand stock when an order
arrives

After an order arrives, the expected on-hand inventory increases to Q+S and is reduced to a value of S on the average just before the next order arrives. Therefore, the average on-hand inventory per cycle is:

$$\frac{(Q_m + S_m)}{2} + \frac{S_m}{2} = \frac{Q}{2} + S_m$$

To write \mathbf{S}_{m} in terms of the reorder point, \mathbf{r}_{m} , let us first assume that the lead time τ_{m} is fixed. Let

 $\xi_{\tau}(x;r_{m}) = r_{m} - x$ be the net inventory at the time an order arrives,

where x

is the number of units demanded in lead time $\boldsymbol{\tau}_{\boldsymbol{m}}.$ Then

$$S_m = E_{\tau_m}$$
 [Net inventory] = $\int_0^\infty \xi_{\tau_m}(x; r_m) f(x; \tau_m) dx$

where $f(x; \tau_m)$ = Density function of demand in time τ_m

$$S_{m} = \int_{0}^{\infty} (r_{m} - x) f(x; \tau_{m}) dx = r_{m} \int_{0}^{\infty} f(x; \tau_{m}) dx$$
$$- \int_{0}^{\infty} f(x; \tau_{m}) dx$$

 $S_m = r_m - \mu_m$ where $\mu_m = Expected lead time demand.$

Then
$$\frac{Qm}{2} + S_m = \frac{Qm}{2} + r_m - \mu_m$$

I = Inventory carrying charge

C = Cost of an item

Total holding cost/year = IC $(r_m - \mu_m + \frac{Qm}{2})$

c. Stockout Costs

Let us define
$$\eta_{\tau_m}$$
 $(x;r_m) = \begin{cases} 0 & \text{if } x-r_m < 0 \\ x-r_m & \text{if } x-r_m \ge 0 \end{cases}$

where $\eta_{\tau_m}^{}$ $(x;\tau_m)$ is the number of back orders per cycle.

If $\bar{\eta}_m(r_m)$ = expected number of back orders per cycle, then

$$\bar{\eta}_{m}(r_{m}) = \int_{0}^{\infty} \eta_{\tau_{m}}(x; r_{m}) h(x) dx = \int_{0}^{\infty} (x-r_{m}) h(x) dx$$

=
$$\int_{m}^{\infty} xh(x) dx-r_m H(r_m)$$
 where $H(r_m) = P[X>r_m]$

and h(x) = marginal distribution of leadtime demand.

Therefore, the expected number of back orders/year =

$$\frac{\lambda_{m}}{Q_{m}} \left[r_{m}^{f^{\infty}} xh(x) dx - r_{m} H(r_{m}) \right]$$

and the expected cost of back orders/year =

$$\frac{\Pi_{m} \lambda_{m}}{Q_{m}} \left[r_{m}^{f^{\infty}} xh(x) dx - r_{m} H(r_{m}) \right]$$

All the terms in the average annual variable cost $\boldsymbol{K}_{\boldsymbol{m}}$ have now been found:

$$K_{m} = \frac{\lambda_{m}}{Q_{m}} A_{m} + IC \left[\frac{Q_{m}}{2} + r_{m} - \mu_{m} \right] + \frac{\pi \lambda_{m}}{Q_{m}} \left[r_{m}^{f} \times h(x) dx - r_{m} H(r_{m}) \right]$$

2. System One and System Two

It was stated in the assumptions that System One and also System Two both follow a periodic review policy with an order up to R stockage policy. Since System One and System Two are identical to each other; the equations will be derived only for System One.

For convenience, a period is assumed to be the time between the receipt of two successive orders rather than between the placement of two successive orders. Costs are described as follows:

a. Ordering and Reviewing Costs

J = Reviewing cost/cycle

A = Ordering cost/cycle

Ordering and reviewing cost/year = $\frac{J_1 + A_1}{T_1}$

where T_l is the period length defined in units of years.

b. Inventory Carrying Cost

The expected net inventory just prior to the arrival of an order is $R_1 - \mu_1 - \lambda_1 T_1$, where μ_1 = mean demand during lead time.

The mean rate of demand remains constant over time and the expected demand per period must be the expected

amount ordered, i.e., $\lambda_1 T_1$. If the expected net inventory immediately after the arrival of a procurement is $R_1 - \mu_1$, it is therefore $R_1 - \mu_1 - \lambda_1 T_1$ just prior to the arrival of a procurement.

The expected unit years of storage incurred per period is

$$T_1 \left[\frac{1}{2} (R_1 - \mu_1) + \frac{1}{2} (R_1 - \mu_1 - \lambda_1 T_1)\right] = T_1 \left[R_1 - \mu_1 - \frac{\lambda_1 T_1}{2}\right]$$

and average inventory carrying cost/year = IC $[R_1 - \mu_1 - \frac{\lambda_1^T 1}{2}]$.

c. Stockout Costs

First we assume the case where the procurement lead time is constant τ . An order placed at time t will arrive in the system at time t + τ , and the next procurement will arrive in the system at time t + τ + T_1 . After the order is placed at time t, the inventory position of the system is R_1 . It is necessary to compute the expected number of back orders occurring between t + τ and t + τ + T_1 . A back order will occur in this period under assumption c if and only if the demand in the time period τ + T_1 exceeds R_1 . Assumption c also assures that after the arrival of the order placed at time t, there will be no remaining back orders, and therefore they must all occur between times t + τ and t + τ + T_1 . Consequently the expected number of back orders incurred per period is

$$R_1^{\int_{0}^{\infty} (x-R_1)} f(x; \tau + T_1) dx$$
 where

f(x; τ + T_1) = Demand distribution during time τ + T_1 .

When lead time is random with density g (τ_1)

with τ_{min} and τ_{max} being lower and upper limits respectively and τ_1 and τ_2 , the lead times for the orders placed at times t and t + T_1 , respectively, the expected number of back orders incurred per period is:

$$=_{R_1^{f^{\infty}}}(x-R_1) \hat{h}(x; T_1) dx$$

where $\hat{h}(x; T_1) = \int_{\min}^{T_{\max}} f(x; \tau_2 + T_1) g(\tau_2) d\tau_2$ which is the demand distribution during time $\tau_2 + T_1$ when lead time is a random variable with density function $g(\tau_2)$. The average number of back orders incurred per year is;

 $E_1(R_1,T_1) = \frac{1}{T_1} R_1^{\infty} (X-R_1) \hat{h}(X; T_1) \text{ dx and the average}$ back order cost per year equals to $\Pi_1 E_1(R_1,T_1)$ and $\bar{n}_1(r) = T_1 E_1(R_1,T_2) \text{ is the expected number of back orders }$ per period.

Finally, the annual variable cost of System One

is:

$$K_1 = \frac{L_1}{T_1} + IC \left[R_1 - \mu_1 - \frac{\lambda_1 T_1}{2}\right] + \Pi_1 E_1 \left(R_1, T_1\right)$$
 and

likewise the annual variable cost of System Two is:

$$K_2 = \frac{L_2}{T_2} + IC \left[R_2 - \mu_2 - \frac{\lambda_2 T_2}{2}\right] + \Pi_2 E_2 \left(R_2, T_2\right)$$

where Li = $J_i + A_i$.

3. Objective Function and Constraints

The objective function of the model consists of the total annual variable costs of each system. Therefore;

 $K = K_m + K_1 + K_2$ which is equal to the minimization

of;

$$\begin{split} \kappa &= \frac{\lambda_m}{Q_m} \quad A_m \, + \, \, \text{IC} \, \left[\frac{Q_m}{2} \, + \, r_m \, - \, \mu_m \right] \, + \, \frac{\Pi_m \lambda_m}{Q_m} \, \, \overline{\eta}_m \, \, (r) \, + \, \frac{L_1}{T_1} \\ &+ \, \, \text{IC} \, \left[R_1 \, - \, \mu_1 \, - \, \frac{\lambda_1 T_1}{2} \right] \, + \, \frac{\Pi_1}{T_1} \, \, \overline{\eta}_1 \, (r) \, + \, \frac{L_2}{T_2} \, + \, \, \text{IC} \, \left[R_2 - \mu_2 - \frac{\lambda_2 T_2}{2} \right] \\ &+ \, \frac{\Pi_2}{T_2} \, \, \overline{\eta}_2 \, \, (r) \end{split}$$

subject to
$$\frac{\lambda_m}{Q_m} \bar{\eta}_m(r) + \frac{1}{T_1} \bar{\eta}_1(r) + \frac{1}{T_2} \bar{\eta}_2(r) \le b$$

where b is the specified total maximum number of back orders per year for the entire system.

The problem at hand is to calculate the optimum values of Q_m , r_m , R_1 , T_1 , R_2 , T_2 . Since the objective function and the constraint are non-linear functions of the

decision variables, we solve the problem using the Lagrange multiplier approach.

4. Optimum Values of Operating Variables

After including the Lagrange multiplier in the formulation, the new objective function becomes:

Minimize

$$\begin{split} \mathbf{L} &= \frac{\partial_{m}}{Q_{m}} \, \mathbf{A}_{m} + \mathbf{IC} \big[\frac{Q_{m}}{2} + \mathbf{r}_{m} - \mu_{m} \big] \, + \, \frac{\Pi_{m} \lambda_{m}}{Q_{m}} \, \bar{\eta}_{m}(\mathbf{r}) \, + \, \frac{L_{1}}{T_{1}} \\ &+ \, \mathbf{IC} \big[\mathbf{R}_{1} - \mu_{1} - \frac{\lambda_{1} T_{1}}{2} \big] \, + \, \frac{\Pi_{1}}{T_{1}} \, \bar{\eta}_{1}(\mathbf{r}) \, + \, \frac{L_{2}}{T_{2}} \, + \, \mathbf{IC} \big[\mathbf{R}_{2} - \mu_{2} - \frac{\lambda_{2} T_{2}}{2} \big] \\ &+ \, \frac{\Pi_{2}}{T_{2}} \, \bar{\eta}_{2}(\mathbf{r}) \, - \, \theta \, \big[\, (\frac{\lambda_{m}}{Q_{m}} \, \bar{\eta}_{m}(\mathbf{r}) \, + \, \frac{1}{T_{1}} \, \bar{\eta}_{1}(\mathbf{r}) \, + \, \frac{1}{T_{2}} \, \bar{\eta}_{2}(\mathbf{r}) \,) \, - \, b \big] \end{split}$$

where θ is the Lagrange multiplier. It is through this Lagrange multiplier that the three systems are linked together mathematically.

The optimum values of the unknown variables can be found by taking the derivatives of the objective function with respect to Q_m , R_m , R_1 , T_1 , R_2 , T_2 , θ ; equating them to zero; solving the equations simultaneously; and ensuring the Kuhn-Tucker conditions are satisfied.

The derivatives were taken with respect to $\mathbf{Q_m}$, $\mathbf{r_m}$, $\mathbf{R_1}$, $\mathbf{R_2}$, θ . A different procedure was utilized to find $\mathbf{T_1}$ and $\mathbf{T_2}$.

The derivatives:

$$\begin{split} & L_{Q_{m}} = \frac{\partial L}{\partial Q_{m}} = -\frac{\lambda_{m} A_{m}}{Q_{m}^{2}} + \frac{IC}{2} - \frac{\lambda_{m}}{Q_{m}^{2}} \, \overline{\eta}_{m}(r) \, (\Pi_{m} - \theta) = 0 \\ \\ & L_{r_{m}} = \frac{\partial L}{\partial r_{m}} = IC - \frac{\lambda_{m}}{Q_{m}} \, \widehat{H}_{m}(r) \, (\Pi_{m} - \theta) = 0 \\ \\ & L_{R_{1}} = \frac{L}{R_{1}} = IC - \frac{1}{T_{1}} \, H_{1}(R_{1}, T_{1}) \, (\Pi_{1} - \theta) = 0 \\ \\ & L_{R_{2}} = \frac{\partial L}{\partial R_{2}} = IC - \frac{1}{T_{2}} \, H_{2} \, (R_{2}, T_{2}) \, (\Pi_{2} - \theta) = 0 \\ \\ & L_{\theta} = \frac{\partial L}{\partial \theta} = \left[(\frac{\lambda_{m}}{Q_{m}} \, \overline{\eta}_{m}(r) \, + \frac{1}{T_{1}} \, \overline{\eta}_{1}(r) \, + \frac{1}{T_{2}} \, \overline{\eta}_{2}(r)) \, - \, b \right] = 0 \end{split}$$

The Kuhn-Tucker conditions:

Therefore the following Kuhn-Tucker conditions must be satisfied:

$$-\frac{\lambda_{m}^{A} M}{Q_{m}^{2}} + \frac{IC}{2} - \frac{\lambda_{m}}{Q_{m}} \bar{\eta}_{m}^{(r)} (\Pi_{m} - \theta) \ge 0$$

IC
$$-\frac{\lambda_m}{Q_m} \hat{H}_m(r) (\Pi_m - \theta) \ge 0$$

IC
$$-\frac{1}{T_1} \hat{H}_1(R_1, T_1) (\Pi_1 - \theta) \ge 0$$

IC -
$$\frac{1}{T_2}$$
 \hat{H}_2 (R_2, T_2) $(\Pi_2 - \theta) \ge 0$

$$-\frac{\lambda_{m}A_{m}}{Q_{m}} + \frac{IC Q_{m}}{2} - \frac{\lambda_{m}}{Q_{m}} \bar{\eta}_{m}(r) (\Pi_{m} - \theta) = 0$$

$$IC r_m - \frac{\lambda_m r_m}{Q_m} \hat{H}_m(r) (\Pi_m - \theta) = 0$$

IC
$$R_1 - \frac{R_1}{T_1} \hat{H}_1 (R_1, T_1) (\Pi_1 - \theta) = 0$$

IC
$$R_2 - \frac{R_2}{T_2} \hat{H}_2 (R_2, T_2) (\Pi_2 - \theta) = 0$$

$$Q_{m} \geq 0$$

$$R_1 \geq 0$$

$$R_2 > 0$$

$$\frac{\lambda_{m}}{Q_{m}} \, \overline{\eta}_{m}(r) \, + \, \frac{1}{T_{1}} \, \overline{\eta}_{1}(r) \, + \, \frac{1}{T_{2}} \, \overline{\eta}_{2}(r) \, - \, b \, \leqslant \, 0$$

$$\theta \left[\frac{\lambda_{m}}{Q_{m}} \, \bar{\eta}_{m}(r) + \frac{1}{\bar{T}_{1}} \, \bar{\eta}_{1}(r) + \frac{1}{\bar{T}_{2}} \, \bar{\eta}_{2}(r) - b \right] = 0$$

From these equations and conditions the optimum values of the unknown variables can be simplified to the following form and the exact values can be found by solving these equations simultaneously:

$$\hat{H}_{m}(r) = \frac{IC Q_{m}}{(\Pi_{m} - \theta) \lambda_{m}}$$

$$Q_{m} = \sqrt{2\lambda_{m}[A_{m} + \overline{\eta}_{m} (\Pi_{m} - \theta)]}$$

$$\hat{H}_1$$
 (R₁, T₁) = $\frac{IC T_1}{(\Pi_1 - \theta)}$

$$\hat{H}_2$$
 (R₂, T₂) = $\frac{IC T_2}{(\Pi_2 - \theta)}$

The procedure to find T_1 and T_2 and subsequently R_1 and R_2 is through iteration and trial and error. This is not unreasonable. In realistic cases other considerations not modelled here usually dictate the length of a period. In most cases the systems control a large number of different items and the same period length is used for each item. Thus, the period length is usually some convenient calendar or financial period, such as a month or a quarter. The procedure we use is shown in the following example.

E. EXAMPLE PROBLEM

1. Additional Assumptions

For this example it is assumed that each system has Poisson arrivals independent of each other. The customers

demand only one item at a time. System One and System Two are resupplied only from the main system and the main system is resupplied from outside suppliers. To be more realistic it is also assumed that the procurement lead times have a gamma distribution with different mean and variances.

The λ 's will be assumed daily demand or arrival rate per day rather than yearly values.

2. Input Values

Before providing the input values, some clarifications must be provided regarding the demand at the main system. When it is stated that $\lambda_{\rm m}$ = 4/day, $\lambda_{\rm 1}$ = 3/day and $\lambda_{\rm 2}$ = 1/day. These represent the mean number of demands which arrive each day directly at the respective systems. However, since all demands at Systems One and Two eventually must filter up to the main system, the cumulative demand at the main system has an expected value of $\lambda_1 + \lambda_2 + \lambda_m = 8$. The main system supports not only Systems One and Two, but also has its own customer demands. The model assumes that the demand at the main system is the superposition of the direct demands at the main system and Systems One and Two. However, since the actual demands placed on the main system in the lower echelon are batched, the demand variability is much greater than would be expected by the superposition process. We will evaluate the seriousness of our assumption about the demand process at the main system with the simulation model described in the next chapter.

In this example the input values for each system are;

- a. Main System
 - $\lambda_{\rm m}$ = 8/day (considering the individual customer arrivals and the remainder being Systems One and Two average demands)

C = \$50/item

I = 0.23

 $\Pi_{\rm m} = 5

 $A_{\rm m} = 500

b. System One

 $\lambda_1 = 3/\text{day}$

C = \$50/item

I = 0.23

1 = \$5

 $A_1 = 100

J₁ = \$100

c. System Two

 $\lambda_2 = 1/\text{day}$

C = \$50/item

I = 0.23

 $\Pi_2 = 5

 $A_2 = 100

 $J_2 = 100

The total system is expected not to exceed

b = 175 total number of back orders at the end of a year.

3. Solution Procedure and Results

It was assumed that for each system, procurement lead times are gamma distributed random variables.

The gamma distribution is defined as:

$$g(t) = \frac{1}{\Gamma(\alpha) \beta^{\alpha}} t^{\alpha-1} e^{-\frac{t}{\beta}}$$
 with

 $\mu = \alpha \cdot \beta$ and

$$Var = \alpha \cdot \beta^2$$

We need the lead-time demand distribution for the case in which lead times are gamma distributed and the process generating demands is Poisson. This is derived below.

The Poisson distribution is

 $f(x) = \frac{(\lambda t)^{x} e^{-\lambda t}}{x} \quad \text{and the demand distribution}$ during lead time is:

$$f(x;\tau) = \int_{0}^{\infty} f(x) g(t) dt$$

$$= \int_{0}^{\infty} \frac{(\lambda t)^{x} e^{-\lambda t}}{x} \frac{t^{\alpha-1} e^{-t/\beta}}{\Gamma(\alpha)^{\beta \alpha}} dt$$

$$= \frac{\lambda^{x}}{\Gamma(\alpha)^{\beta \alpha} x!} \int_{0}^{\infty} t^{\alpha-1} + x e^{-(\frac{1}{\beta} + \lambda)t} dt$$

$$= \frac{\lambda^{\mathbf{X}}}{\Gamma(\alpha)} \frac{\Gamma(\alpha+\mathbf{X})}{\beta^{\alpha} \mathbf{X}!} \frac{\Gamma(\alpha+\mathbf{X})}{(\frac{1}{\beta}+\lambda)^{\alpha+\mathbf{X}}} \int_{0}^{\infty} \frac{(\frac{1}{\beta}+\lambda)^{\alpha+\mathbf{X}}}{\Gamma(\alpha+\mathbf{X})} e^{-(\frac{1}{\beta}+\lambda)\mathbf{t}} d\mathbf{t}$$

$$\int_{0}^{\infty} \frac{(\frac{1}{\beta}+\lambda)^{\alpha+\mathbf{X}}}{\Gamma(\alpha+\mathbf{X})} \frac{e^{-(1+\mathbf{X})}}{\Gamma(\alpha+\mathbf{X})} \frac{e^{-(\frac{1}{\beta}+\lambda)\mathbf{t}}}{(\frac{1}{\beta}+\lambda)^{\alpha+\mathbf{X}}} d\mathbf{t} = 1 \quad \text{so}$$

$$f(\mathbf{X};\tau) = \frac{\lambda^{\mathbf{X}}}{\Gamma(\alpha)} \frac{1}{\beta^{\alpha} \mathbf{X}!} \frac{\Gamma(\alpha+\mathbf{X})}{(\frac{1}{\beta}+\lambda)^{\alpha+\mathbf{X}}} = \frac{\Gamma(\alpha+\mathbf{X})}{\Gamma(\alpha)} \frac{1}{\beta^{\alpha}} \frac{$$

In fact this is a Negative Binomial distribution with

$$\rho = \frac{1/\beta}{1/\beta + \lambda} .$$

The parameters α and β for each system are:

$$\alpha_{\rm m} = 12.8$$
 $\alpha_{\rm 1} = 5.43$ $\alpha_{\rm 2} = 2.5$ $\beta_{\rm m} = 3.125$ $\beta_{\rm 1} = 4.6$ $\beta_{\rm 2} = 6.$

The main system has a Negative Binomial distributed lead time demand with parameters:

$$\mu_{\rm m} = 8 \cdot (12.8) (3.125) = 320$$

$$var_{\rm m} = 8 \cdot (12.8) (3.125) [1+8 \cdot (3.125)] = 8320$$

System One has the parameters:

$$\mu_1 = 3 \cdot (5.43) \cdot (4.6) = 74.934$$

$$var_1 = 3 \cdot (5.43) \cdot (4.6) \cdot [1+3(4.6)] = 1109.0232$$

System Two has the parameters:

$$\mu_2 = 1 \cdot (2.5) \cdot 6 = 15$$

$$var_2 = 1 \cdot (2.5) \cdot 6 [1+1(6)] = 105$$

Because the negative binomial is computationally intractable, we use the normal approximation for the calculation of lead time demand probabilities. The normal distributions are assumed to have the same mean and variance as the negative binomial distributions they replace.

To solve this example problem, it is necessary to make an initial estimate for the Lagrange multiplier θ

which must be less than or equal to zero. After the initial estimate if the total number of back orders per year exceeds b, then the absolute value of θ should be increased gradually until the number of back orders converges to b.

For this problem θ = -1.5 works very well. For the main system the formulas are:

$$Q_{m} = \sqrt{2\lambda_{m}[A_{m} + \bar{\eta}_{m}(r)(\bar{\Pi}_{m} - \theta)]/IC}$$

$$H_{m}(r) = \frac{Q_{m} IC}{(II_{m} - \theta) \lambda_{m}} = P [X > r_{m}]$$

 $\mathbf{Q}_{\mathbf{m}}$ is calculated to be 550 and

$$H_{m}(r) = P[X > r_{m}] = \frac{550 (0.23) \cdot 50}{(5+1.5) 2920} = 0.3332455216$$

From the inverse standard normal distribution TI-59 calculator program (Appendix G), the reorder level at the main system is found to be:

$$r_{\rm m} = \sigma_{\rm m} \cdot (0.4305333395) + \mu_{\rm m} = 359.2706827$$

$$\vec{\eta}_{m}(\mathbf{r}) = (\mu_{m} - \mathbf{r}_{m}) H_{m}(\mathbf{r}) + \sigma_{m} \mathcal{O}(\frac{\mathbf{r}_{m} - \mu_{m}}{\sigma_{m}})$$

where \emptyset is the functional value of standard normal distribution at 0.4305333395

$$\bar{\eta}_m(r)$$
 = 20.08139334. Using this value in the Q_m

formula yields

$$Q_{m} = \sqrt{2.2920 \left[500 + 20.08139334 \left(5 + 1.5\right)\right]} = 568.84$$

which is not equal to the first estimated value. If $\mathbf{Q}_{\mathbf{m}}$ is taken 569 then:

$$H_{m}(r) = \frac{569 (0.23) \cdot 50}{(5 + 1.5) 2920} = 0.3447576396$$

$$r_{m} = \sigma_{m}$$
 (0.399071201) + $\mu_{m} = 356.4008941$

$$\tilde{\eta}_{m}(r) = 21.05458646$$

Using this value in the $\mathbf{Q}_{\mathbf{m}}$ formula yields

$$Q_{m} = \sqrt{2.2920 \left[500 + 21.05438646 \left(5 + 1.5\right)\right]} = 568.69$$

$$(0.23) \cdot 50$$

which is very close to the initial $Q_{\overline{m}}$ value. A summary of the results for the main system follows:

$$Q_m = 569$$

$$s_m = 36.400891$$

$$\bar{\eta}_{m}(r) = 21.05438646/period$$

$$\bar{\eta}_{m}(r) = 108.047115/year$$

$$K_{\rm m} = $6796.50/{\rm year}$$

To solve for the optimum values for Systems One and
Two a different procedure is followed. The total annual cost
of Systems One and Two is a convex function of the period length
T. For different values of T there are different total annual

cost values. The minimum of these values is the optimum total annual variable cost and the corresponding T and R values are the optimum operating values. A TI-59 program was written to perform the line search for the best value of T. The program is found in Appendix G. User information and the features of the program are also in the same appendix. This program evaluates the R value, safety stock, back orders per period, back-orders per year, annual reviewing and ordering cost, annual holding cost, annual back order cost, and finally, the total annual cost.

Using this program the optimum values for Systems
One and Two are:

 $R_1 = 306.809389$

 $T_1 = 2.39 \text{ months } (72.6958 \text{ days})$

 $s_1 = 13.78788901$

 $\bar{n}_1(r) = 8.670289086/period$

 $\bar{\eta}_1(r) = 43.53283223$

 $K_1 = $2634.41/year$

 $R_2 = 134.9807003$

 $T_2 = 4.07 \text{ months } (123.7958333 \text{ days})$

 $S_2 = 0$

 $\bar{\eta}_2(2) = 8.13480099/\text{period}$

$$\bar{\eta}_2(r) = 23.98467122/year$$

The total back orders per year are:

$$\bar{\eta}_{m}(r) + \bar{\eta}_{1}(r) + \eta_{2}(r) = 108.047 + 43.532 + 23.984$$

$$= 175.56$$

which is the same as b = 175.56.

The total annual variable cost is:

$$K = K_m + K_1 + K_2 = 6796.50 + 2634.41 + 1377.56$$

K = \$10808.47.

III. COMPUTER SIMULATION FOR MULTI-ECHELON MULTI-LOCATION SINGLE ITEM INVENTORY SYSTEM

To check the analytical results, a computer simulation was written which uses the same operating assumptions and input parameters that were made for the analytical model. This model, however, simulates the real world more accurately since some of the simplifying assumptions required to obtain analytical results were not necessary in the simulation. Also, the demands placed at the main system from the lower echelon systems were batched as in the real world.

A. DESCRIPTION OF THE SIMULATION MODEL

As in the analytical case, there are three systems in the simulation model; the main system, System One and System Two. The flow charts of this program are in Appendix A.

1. Main System

This system uses a continuous review policy. When the stock level reaches the reorder point, it orders the amount Q. The decision variable for reordering is the inventory position. When an order is placed, the inventory position increases by an amount of Q. If the new inventory position is less than the reorder point, the system places an additional order for another amount of size Q. Then the

inventory position increases with one more Q. This continues until the inventory position exceeds r. The order policy is thus (nQ,r). N is the smallest integer that will make the inventory position higher than the reorder point.

There are independent customer arrivals to the main system and also group demands that are placed by System One and System Two when those systems replenished their stocks at the end of their periods.

The number of units demanded per requisition is one. The number of units demanded for resupply to the lower echelon systems is random depending on the demands at the lower echelon systems and the parameters R_1 , R_2 . The program is capable of allowing geometrically distributed quantities demanded per requisition. Back orders at the main system are satisfied by filling the back orders to individual customers first followed by filling any back orders due to System One and System Two on a first-come, first-served basis.

When a demand occurs from the lower echelon systems for which there is not sufficient on-hand inventory at the main system, the maximum amount is filled and the rest is put into the back order queue. As soon as a shipment arrives at the main system, the back orders are filled. The times between the customer arrivals are independent of each other and exponentially distributed. The lead times are also independent of each other and gamma distributed.

2. System One and System Two

Both systems operate identically. The only difference might be in the values of the system parameters. They have the same periodic review policy which is: order up to R at each review time. The program is also capable of making a decision at each review time regarding the placement of an order if the inventory position is less than or equal to a threshold value, is an (r,R) policy. If r is taken to be R-1, then the system orders up to R at each review time even if there is only one demand in a period.

The times between customer arrivals are independent of each other and exponentially distributed. The number of units demanded per requisition is one. The lead times are independent of each other and gamma distributed.

B. HOW TO USE THE PROGRAM

Two simulations are given in Appendix C and Appendix D. The programs differ primarily in the type of output that is generated. The program given in Appendix C produces a Versatec plot output showing the inventory position for the main system and the lower echelon systems. A simulated period of up to 4 or 5 years can be run with a Class K job.

On the Versatec output for each system, there are two plots: one plot has a small triangle at each point representing the inventory position. The second plot represents the net inventory. The first figure shows the main system

values, the second figure shows the System One values and the third figure shows System Two values all plotted versus time.

The second program can be used to simulate operations over arbitrarily long periods of time. There are no dimension restrictions. This program gives the net inventory at the end of each period for all the systems; total demand in the periods of both System One and System Two; demand during a lead time for the main system; average on-hand inventory; and average number of items short per day for each system.

The input required by the programs is described by the variable definition list given in Appendix H. The programs use the IMSL subroutines for random number generation.

C. RUNS

1. Starting Conditions

There were two different starting conditions entered for the main system. In the first case the inventory position and net inventory were set initially to equal the order quantity. In the second case, the inventory position and the net inventory were set equal to the reorder point plus the order quantity. For Systems One and Two the inventory positions and net inventories were set initially at R_i.

2. Results of Runs

In all runs the length of time was 10 years. average yearly results were obtained by dividing the results for the 10 year period by 10.

Using the policy parameters determined by the mathematical model and the first set of starting conditions the following simulated results were obtained.

a. Main System

The number of orders in 10 years = 51.

The number of back orders in 10 years = 3026.

The average on-hand inventory over 10 years =

313.026.

The average safety stock when an order arrives = 56.54 units.

The average annual costs are:

Ordering costs: \$2,550.00

Holding costs: 3,599.80

Back Order Costs: 1,513.00

Total Cost \$7,662.80

System One

Number of back orders in 10 years = 587.

The average on-hand inventory over 10 years = 114.482 units.

The average annual costs are:

Ordering and Reviewing Costs: \$1,004.18

Holding Costs: 1,316.54

Back Orders Costs: 293.50

Total Costs \$2,614.22

c. System Two

Number of back orders in 10 years = 287.

The average on-hand inventory over 10 years = 57.618 units.

The average annual costs are:

Ordering and Reviewing Costs: \$ 589.68

Holding Costs: 662.61

Back Orders Cost: 143.50

Total Costs \$1,395.79

(1) Total results

The total number of back orders for the whole system per year is 302.6 + 58.7 + 28.7 = 390.

The total annual variable cost for the whole system is \$7,662.80 + \$2,614.22 + \$1,395.79 = \$11,672.81.

The model was run again with the second set of starting conditions and the same set of policy parameter values. The simulation results are summarized below:

(a) Main System

The number of orders in 10 years = 50.

The number of back orders in 10 years = 1665.

The average on-hand inventory over 10 years = 346.937.

The average safety stock when an order arrives = 77.46.

The average annual costs are;

Ordering Costs: \$2,500.00

Holding Costs: 3,989.78

Back Order Costs: 832.50

Total Costs: \$7,322.28

(b) System One

Number of back orders in 10 years = 582.

The average on-hand inventory over

10 years = 118.696 units.

The average annual costs are;

Ordering and Reviewing Costs: \$1,004.18

Holding Costs: 1,365.00

Back Orders Costs: 291.00

Total Costs: \$2,660.18

(c) System Two

Number of back orders in 10 years: 314.

The average on-hand inventory over 10 years =

57.401 units.

The average annual costs are;

Ordering and Reviewing Costs: \$ 589.68

Holding Costs:

660.11

Back Orders Costs:

157.00

Total Costs:

\$1,406.79

(2) Total Results

The total number of back orders for whole system per year is 166.5 + 58.2 + 31.4 = 256.

The total annual variable cost for whole system is \$7,322.28 + \$2,660.18 + \$1,406.79 = \$11,389.25.

3. Comparison of Both Starting Conditions

It is obvious that if both results are compared there is a considerable decrement on annual back orders but not a great difference in annual variable costs.

$$\frac{390-256}{390} = 0.3436$$

$$\frac{11672.81 - 11389.25}{11672.81} = 0.02429$$

IV. COMPARISON OF ANALYTICAL AND COMPUTER SIMULATION RESULTS

By comparing the analytical results with the simulation results, it is possible to evaluate the analytical model. If the measures of effectiveness predicted by the analytical model are reasonably close to those generated by the simulation, there is support for the analytical results. In the table below we compare the main measures of effectiveness: back orders and costs.

	Total Back Orders Per Year	Total Variable Costs Per Year
Analytical result	175.56	\$10,808.17
Simulation result (For both starting conditions)	390.00	\$11,672.81 or
	256.1	\$11,389.25

This table shows that the average yearly back orders are considerably higher than what is estimated by the mathematical solution. In fact, the total number of back orders generated in the simulation does not even meet the constraint. Since the solutions produced by the simulation model, especially with respect to the number of back orders do not agree with the mathematical model, this suggests that some of the assumptions made in deriving the mathematical results are not reasonable. The major differences between the simulation results and the analytical results are found in the measures

of effectiveness at the main system. The results for the lower echelon track reasonably closely.

The costs are also a little higher than what is expected but much closer percentage wise.

As an explanation for the large differences observed in the number of back orders at the main system with the simulation and analytical models, let us reexamine the assumptions we made that affect back orders. Back orders results from inadequate amounts of safety stock to protect the inventory system against excessively large numbers of demands in a lead time. The safety stock is manipulated by control of the reorder point r. After the reorder point is hit and an order placed, the system is totally at the mercy of the demands that occur during the lead time. If the variance of lead time demand is underestimated, then large stockouts will occur, even if the mean lead time demand is estimated accurately. The lead time demand is affected by two random quantities: (1) the demand distribution and (2) the lead time distribution.

For purposes of making the mathematics tractable, we assumed in our analytical model that demands at the main system flowed in at a smooth continuous rate λ which was taken to be the sum of the rates of demand incurred directly at the main system $\lambda_{\rm m}$ and those which occurred at the lower echelon systems $\lambda_{\rm l}$ and $\lambda_{\rm l}$. The lead times were assumed to

be normally distributed with mean equal to $\lambda \tau$ and variance = $\lambda_{\tau} (1 - \lambda_{\beta})$ where τ is the mean value of the lead time.

Because of the relatively high demand rates used in the example runs, the normal assumption should be justified by the Central Limit Theorem. However, as is illustrated by the Versatec plot output (See Appendix F) the demands at the main system are far from smooth. What happens is the demands which arrive directly at the main system cause the inventory position to drop off smoothly. However, when the replenishment orders from the lower echelon are received, large drops occur in the inventory position of the main system. Recall that the lower echelon systems order in batches once each period from the main system. If the main system simply tries to average out demands (as assumed by the analytical model), it will sometimes have very large amounts of excess stocks when shipments arrive and sometimes very large numbers of back orders. The high variance in lead time demand caused by the irregular demand actually seen at the main system causes the problem. Since all demands eventually flow through the main system and this is assumed by our analytical model, the problem cannot lie in the value used for the mean lead time demand.

Let us explore further what happens when reorders are triggered at the main system. Demands directly at the main system eat away smoothly at the inventory position. Then a

very large quantity, say X, is demanded by one of the lower echelon systems. There is a very good chance that the large order placed by the lower echelon system will trigger a reorder by the main system. However, if the demand causes a large overshoot of the reorder point, the main system may have much less stock to live off of until the shipment arrives. For example, suppose the inventory position is IP = 347, the reorder point is r = 300, and System One places it's resupply order for 200 units. An order will be placed by the main system but instead of having 300 units to keep it going until the order is received, it will have only 147 units. It is clear that if 300 is the amount of stock needed to provide reasonable protection against demands in a lead time, large numbers of back orders would be expected. Effectively, in the example above, the reorder level was not r = 300 but r = 147 and the safety stock negative. impact of this surge in demand caused by the batching of demands received from the lower echelon systems is to reduce the "effective" reorder point from the value r to a value r'<r. Because the actual demand distribution witnessed b, the main system is difficult to describe mathematically, the actual value of r' cannot be determined analytically. However, it is clearly less than r and may be much more so.

In the next chapter we describe a policy modification for operation of the main system that was suggested by the

observations above. The modification attempts to allow the main system to anticipate the surge of demands that will be received by the lower echelon systems.

V. AN ALTERNATIVE SOLUTION

A. DESCRIPTION OF ALTERNATIVE PROCEDURE

Due to the reasons mentioned in Chapter IV, the number of back orders found by simulation were much higher than the number of back orders predicted by the mathematical model. The question is how to run the multi-echelon system so that the large number of back orders seen earlier can be reduced.

Obviously, what needs to be done is to reduce the impact of the very large demands that occur when the lower echelon systems place their resupply orders. With modern day communication and data systems, it would be feasible to allow the main system to "see" every demand that occurs anywhere in the system. If the main system is given the visibility, it will be able to take action to get the stock on its shelves in anticipation of the large demands from the lower echelon systems. The main system can do this if it uses as its reorder point the pseudo inventory position which is like the inventory position except that it decreases only when direct customer demands are encountered at any of the three systems. The pseudo inventory position is unaffected by the batch replenishment demands placed by the lower echelon systems. For example, if a customer requests

a unit from System One, the pseudo inventory position at the main system decreases by one. However, if System One places an order for 200 units at the main system, the pseudo inventory position does not change.

This new policy is referred to in this thesis as the "early warning policy." Clearly, the pseudo inventory position will always reach the reorder point before the inventory position. Therefore, orders will always be placed earlier and consequently, the number of back orders should decrease. The price paid will be in terms of extra holding costs. The results of a simulation using the early warning policy should be more nearly like those of the mathematical model since, in effect, the mathematical model makes the assumption of early warning. Implicitly, assumption of demand which is the superposition of the direct demands occurring at the main system and Systems One and Two is equivalent to the "early warning assumptions."

In the next section, results are given of a simulation of the multi-echelon system with early warning. The flow-chart of the simulation model is given in Appendix B. The actual FORTRAN computer program is given in Appendix E.

B. COMPUTER SIMULATION RESULTS FOR EARLY WARNING POLICY

The same starting conditions and input parameters used
in the previous simulation run were utilized here. The
results are summarized as follows:

1. Main System Results

The number of orders in 10 years = 50.

Total number of back orders in 10 years = 126.

The average on-hand inventory = 488.093.

Average safety stock = 210.96.

The costs are:

Ordering costs = \$2,500.00Holding costs = 5,613.00Back order costs = 63.00Total costs \$8,176.07

2. System One Results

Total number of back orders is 10 years = 665.

The average on-hand inventory over 10 years = 117.846.

The costs are:

Ordering and reviewing costs = \$1,006.18 Holding Costs = 1,355.23Backorder costs = 332.50Total costs \$52,691.91

3. System Two Results

Total number of back orders in 10 years = 312.

The average on-hand inventory over 10 years = 58.552.

The costs are:

Ordering and reviewing costs = \$ 589.68
Holding costs = 673.35
Back orders costs = 156.00
Total costs \$ \$1,419.03

C. COMPARISON OF EARLY WARNING POLICY SIMULATION RESULTS WITH ANALYTICAL AND FIRST SIMULATION RESULTS

The comparison will be done with respect to total yearly back orders and total variable system costs (considering all three system entities).

	Entire System	
	Number of Back Orders Per Year	Total Cost
Analytical Result	175.56	\$10,808.47
First Simulation Results	256.1	11,389.25
Early Warning Simulation Results	110.3	12,287.01

Since the effects of implementing the early warning policy will be observed primarily at the main system, we also produce the results obtained for the main system individually.

	Main System	
	Numbers of Back Orders Per Year	Total Cost
Analytical Result	108.05	\$6,796.50
First Simulation Result	166.50	7,322.28
Early Warning Simulation Results	12.6	8,176.07

This table shows the differences better than the first one. The number of back orders decreases 88.3 percent, simultaneously as the costs increase about 10.44 percent. The reason for the higher cost is because the increment in safety stock increases the carrying costs.

VI. CONCLUSIONS

The differences between the results of the mathematical and the simulation model can be explained largely as a result of the assumptions made about the demand process. In the main system it was assumed that the demand was smooth (the superposition of three Poisson processes), but in the simulation model the actual demand at the main system was as it would be in actual practice. There were batches of demand placed by System One and System Two in addition to the individual customer demands directly at the main system. These demand batches in fact increased the variance in lead time demand beyond that modelled. This explains why many more back orders were generated in the simulation model than what was predicted by the mathematical model.

The simulation model developed in this thesis is useful for making comparisons and examining the effects of policy changes or parameter changes. Moreover, it is one of the best ways to check the reasonableness of all of the simplifying assumptions made in order to obtain analytical solutions.

The mathematical results described in this thesis do not adequately determine the reorder point. The predicted number of stockouts is much less than the simulated numbers. As explained earlier, this is probably due to the assumptions

made in the model about the variance of lead time demand. Further study is needed to determine what could be done in the analytical model to better approximate what happens in actual practice.

The early warning policy discussed in this thesis did provide for great reductions in the number of stockouts system wide. Since stockouts are probably the most important consideration in military supply systems, the early warning policy is recommended, even though the holding costs are larger. Additional study is required to see if the reorder level in the early warning policy can be reduced substantially from the value determined by the mathematical model. Preliminary evidence is that the reorder level can be reduced significantly (25 percent or so) without generating excessively many back orders if the early warning policy is used.

Our simulation models allow us to view the effect of changes but they cannot be used to optimize the values of the policy parameters. For that objective, additional work in the mathematical modelling area is required.

In this thesis we have tried to model a multi-echelon inventory system analytically, by linking together the individual echelons and locations through a single objective function and a constraint on backorders system wide. We, knowingly, were making various simplifying assumptions to facilitate the derivation of solutions. As reported above,

the resulting solution for the reorder level at the main system led to many more back orders than predicted. The other solutions; R_1 , R_2 , Q, T_1 , and T_2 appear to be satisfactory.

In order to model adequately the multi-echelon system, it will probably be necessary to build into the determination of the reorder lead at the main system the values of the parameters R_1 and R_2 at the lower echelon systems. This will be the only way to accurately describe the actual demand process that is observed at the main system. We recommend future work in this area.

APPENDIX A

FLOWCHARTS OF FIRST SIMULATION MODEL

Subroutine One = Ship arrivals to System One

Subroutine Two = Ship arrivals to System Two

Subroutine Three = Ship arrivals to Main System

Subroutine Four = Periodic review of System One

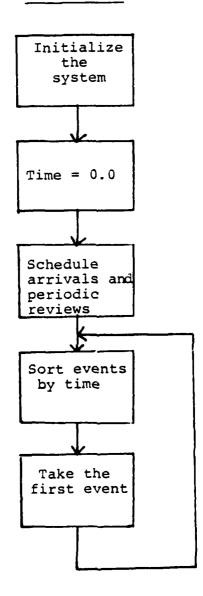
Subroutine Five = Periodic review of System Two

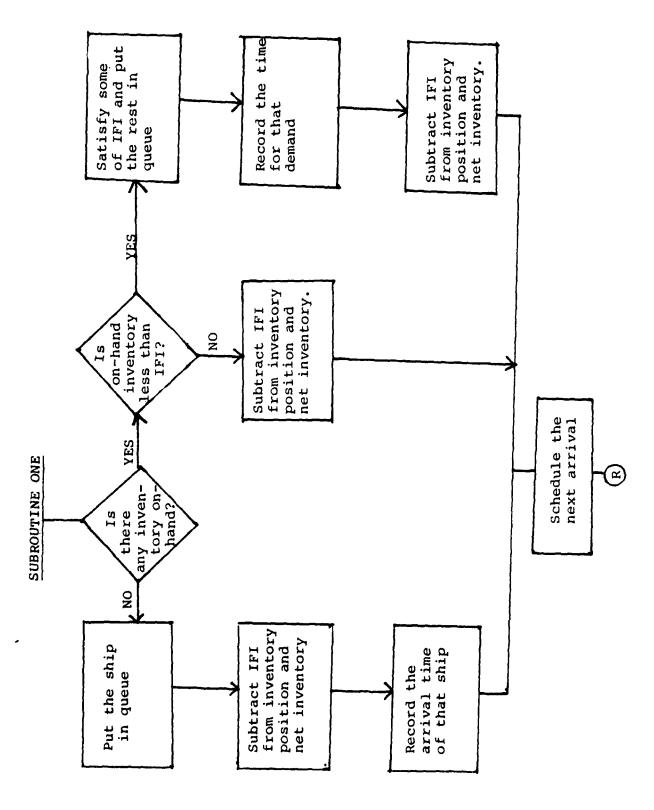
Subroutine Six = Shipment arrival to System One

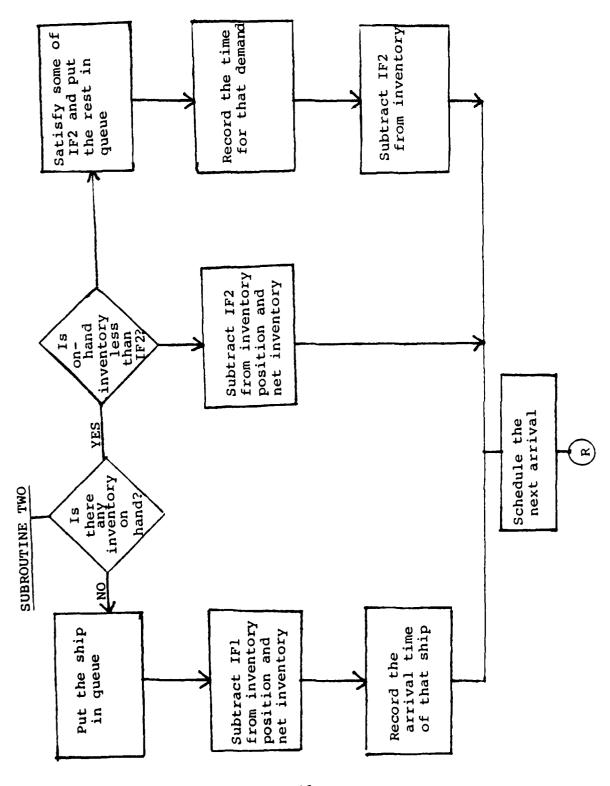
Subroutine Seven = Shipment arrival to System Two

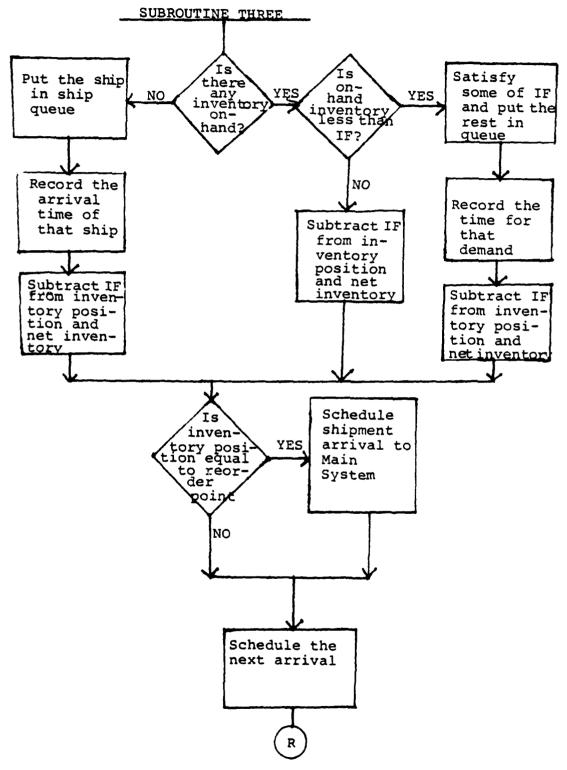
Subroutine Eight = Shipment arrival to Main System

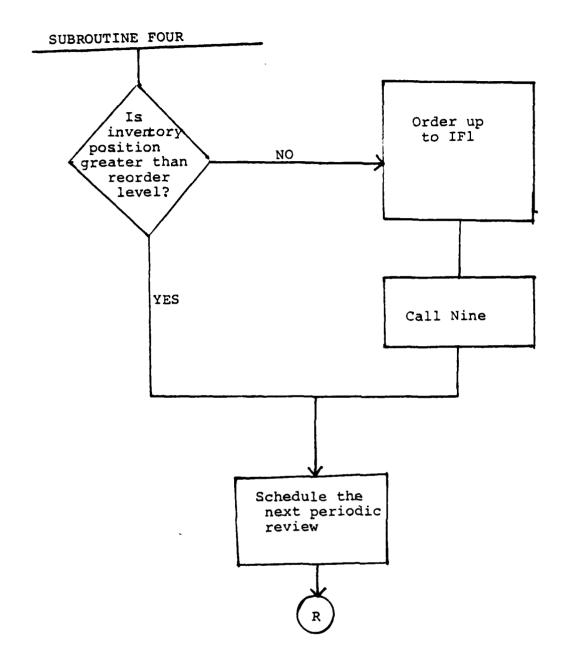
MAIN PROGRAM

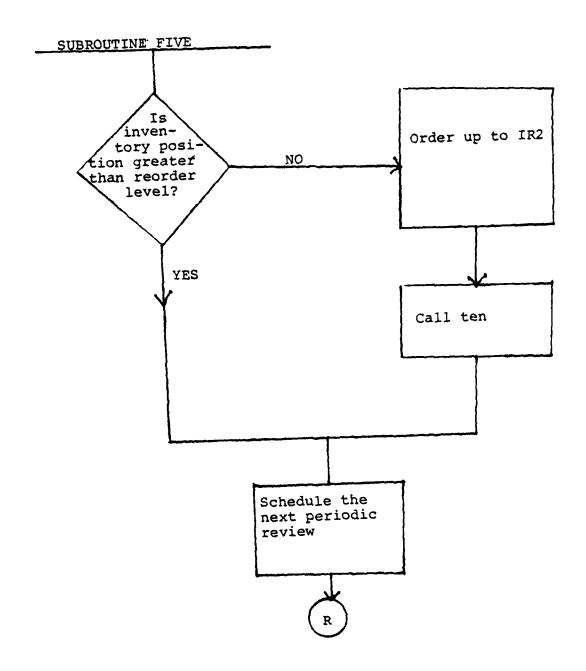


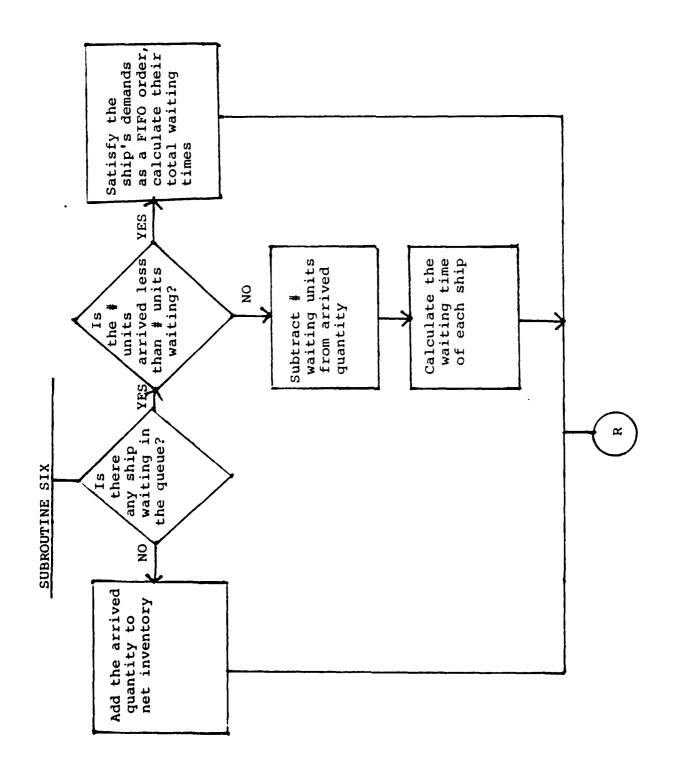


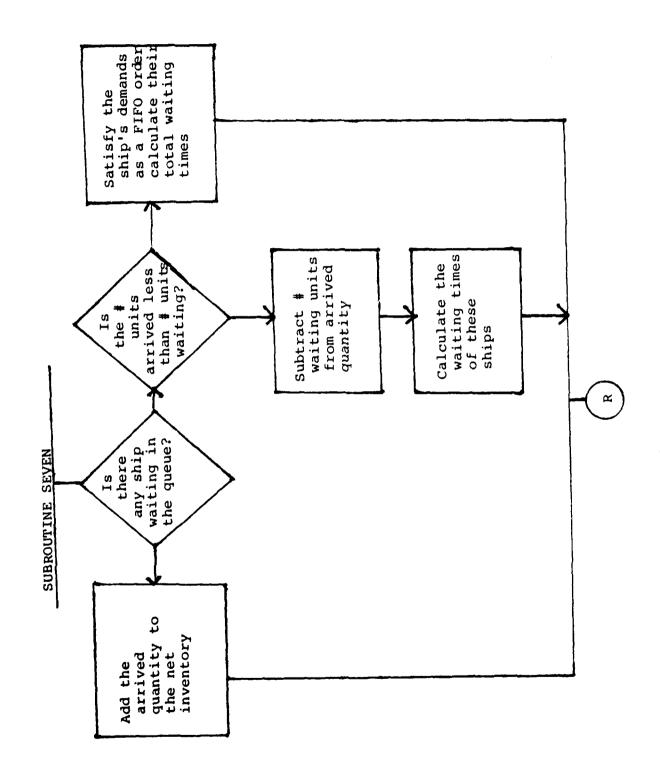


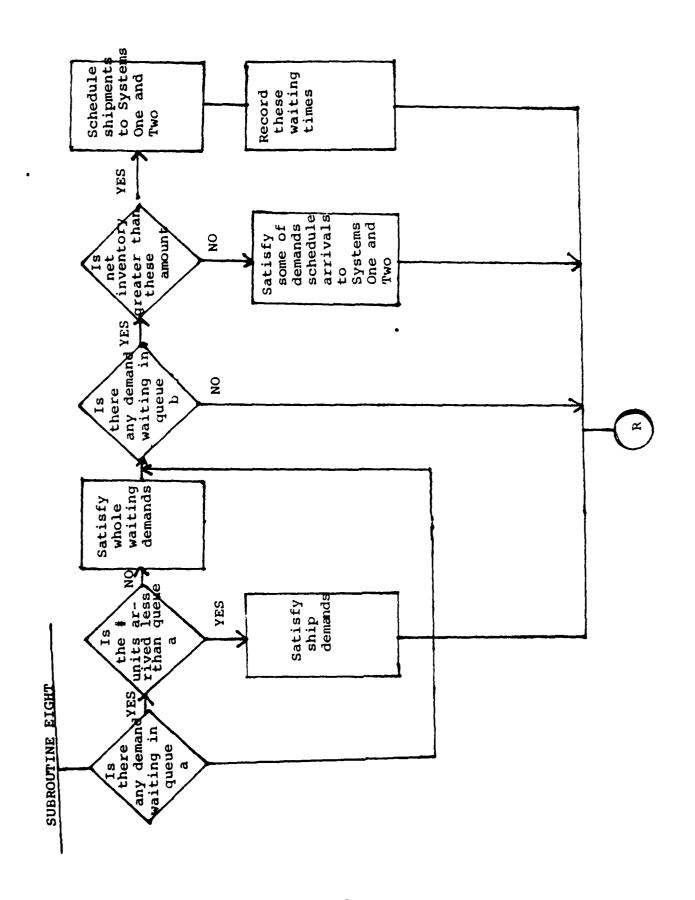


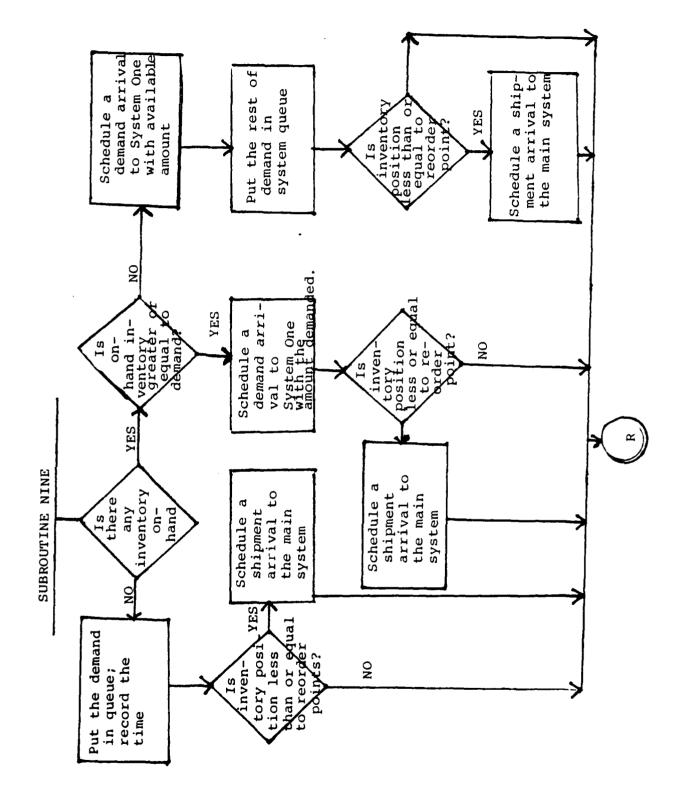


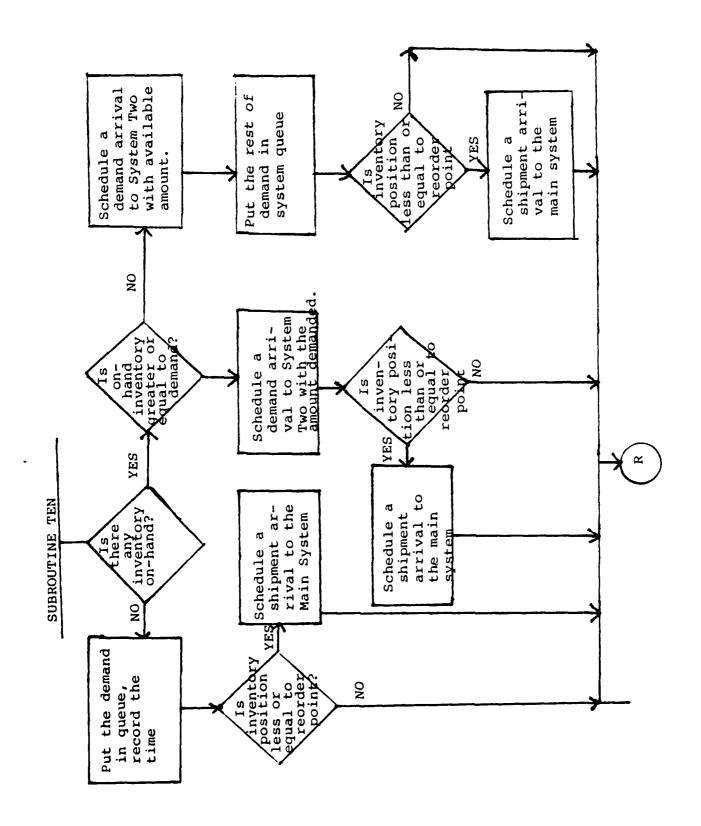












APPENDIK B

Flowcharts of Early Warning Simulation Model

Subroutine One = Ship arrivals to System One

Subroutine Two = Ship arrivals to System Two

Subroutine Three = Ship arrivals to the Main System

Subroutine Four = Periodic review of System One

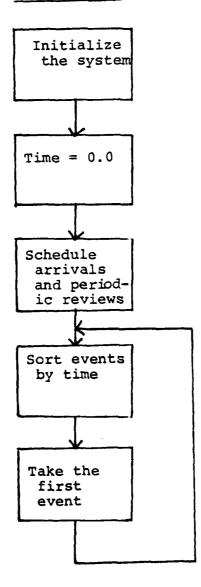
Subroutine Five = Periodic review of System Two

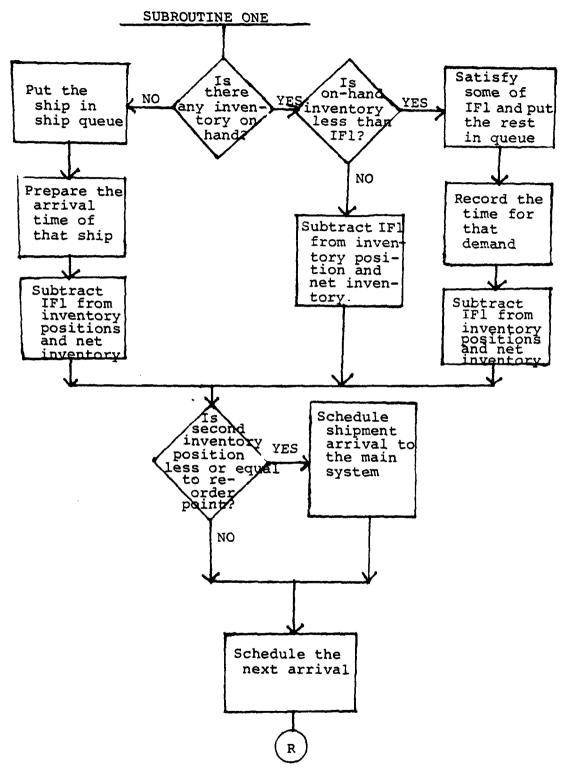
Subroutine Six = Shipment arrival to System One

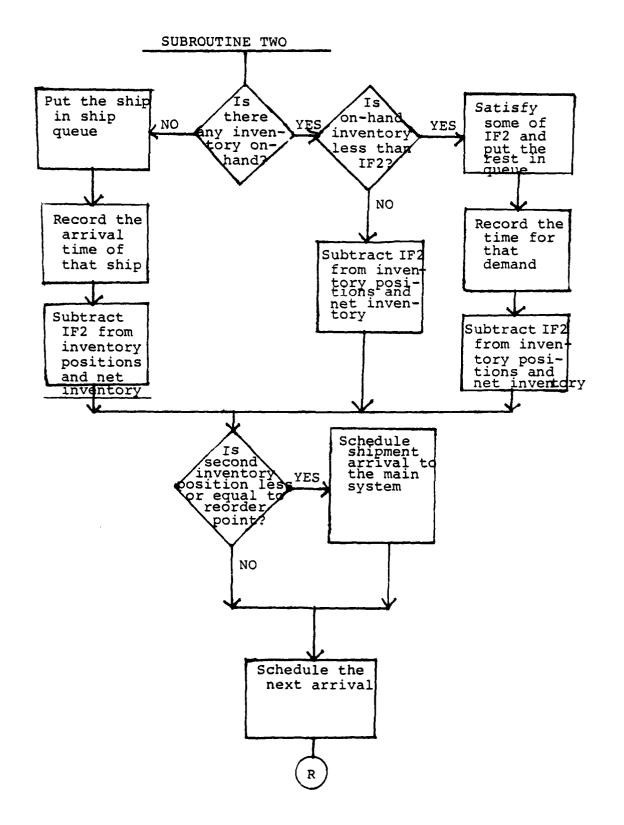
Subroutine Seven = Shipment arrival to System Two

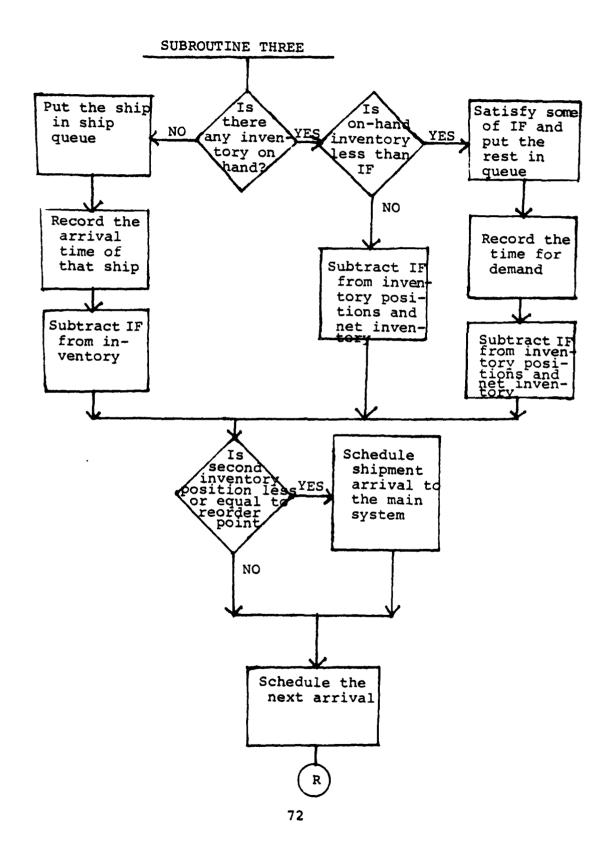
Subroutine Eight = Shipment arrival to the Main System

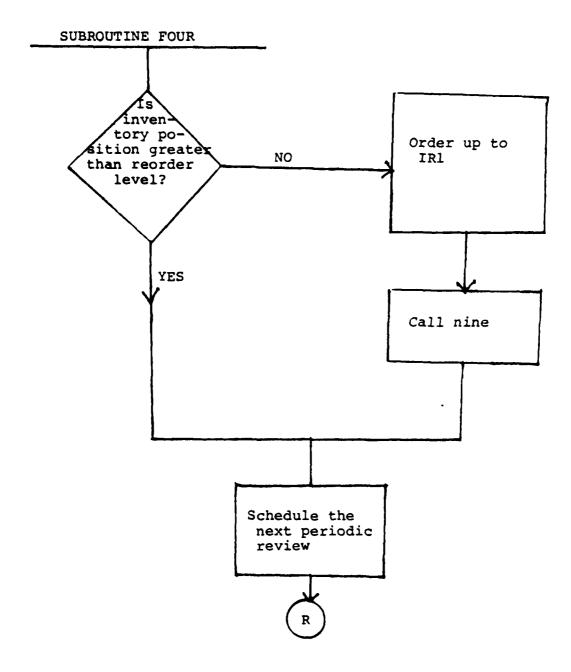
MAIN PROGRAM

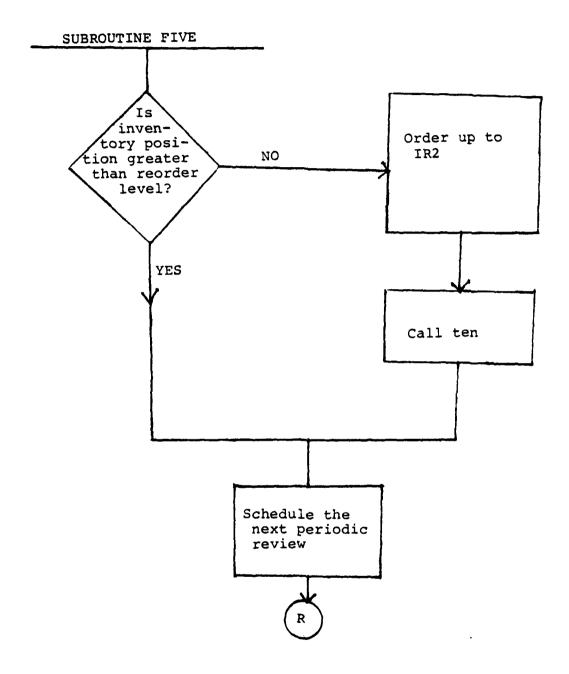


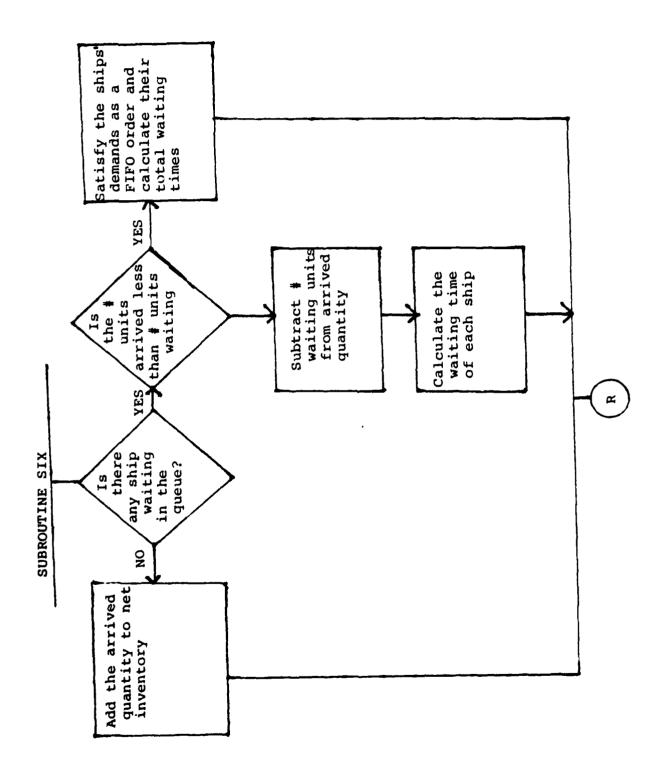


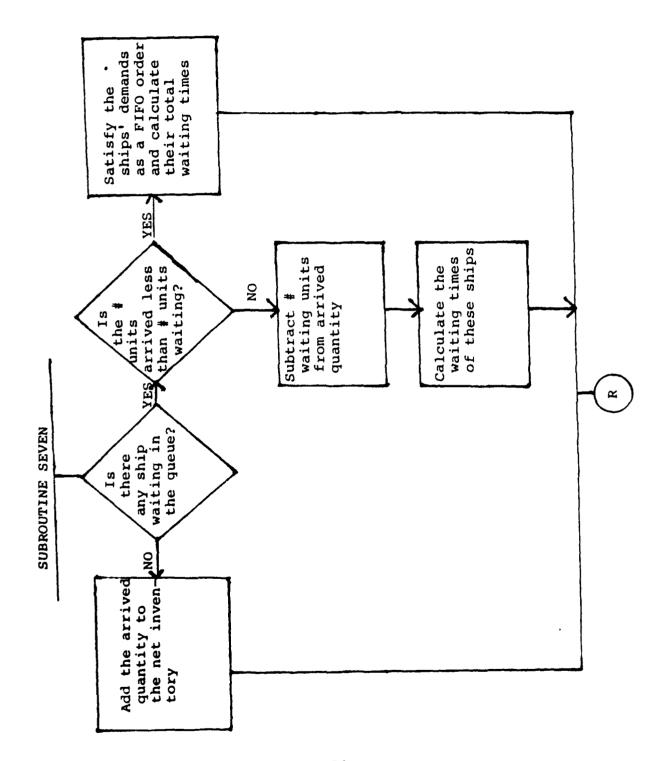


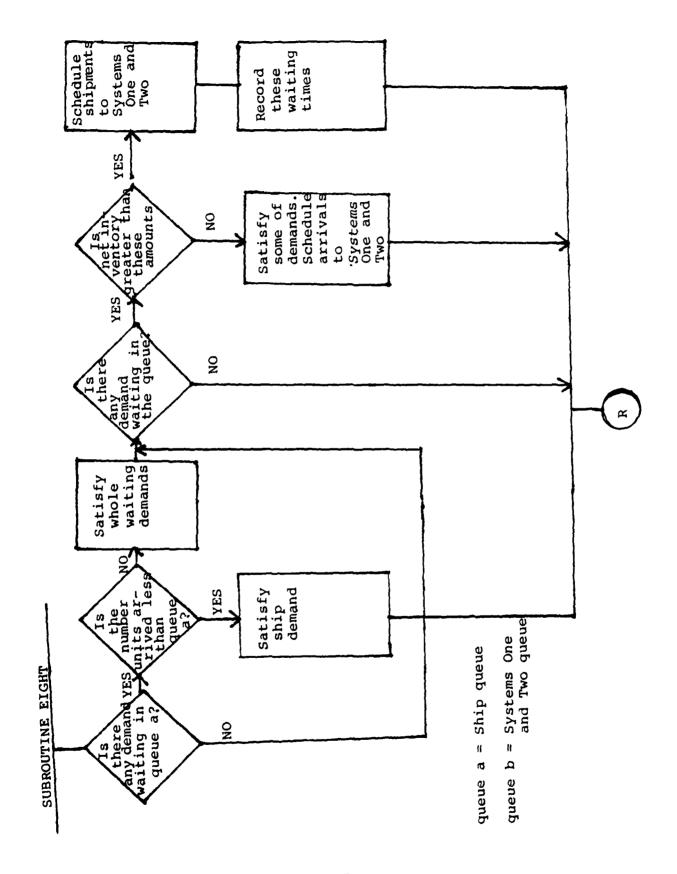


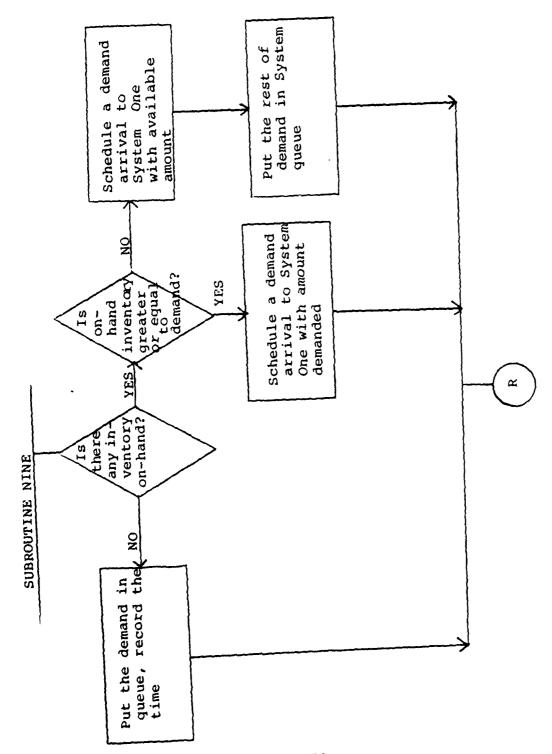


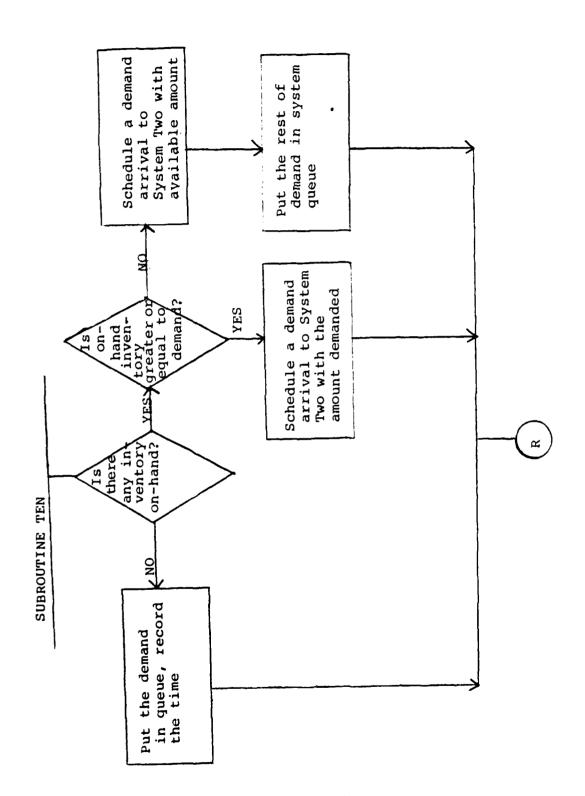












APPENDIX C

Simulation Program; Versatec Plotter

```
OSEE01, DSEED2, DSEED3, DSEED4, DSEED5, DSEED6, DSEED7, DSEED8, DSE
```

```
100 FGRMAT(2110) 18.15

100 FGRMAT(2110) 18.15

110 FGRMAT(2110) 18.15

110 FGRMAT(2110) 18.15

110 FGRMAT(2110) 18.15

110 FGRMAT(2110) 18.15

120 FG
```

```
F(EVENT(II).EQ.99999.) GO TO 10
ONTINUE
                                                                                                                                                                                                                                                    EVENT(II).EQ.99999.) GO TO 12
                                                                                                                                                                                                                                                                                                I=KI
F(EVENT(II).EQ.99999.) GO TO 14
ONTINJE
                                                                                   EVENT(II).EQ.99999.) GO TO 6
F(EVENT(II).EQ.99999.) GO TO
                        ) =T IME+S1(1)
I )=1
                                                                                                                                                                        NI(II)=TIME+S3(1)
ENI(II)=3
9 KI=1,NM
                                                                                                   ) =T IME+S2(1)
I)=2
                                                                                                                                                                                                                       (II)=TIME+TI
IT(II)=4
KI=1,NM
                                                                                                                                                                                                                                                                      I(II)=TIME+T2
VI(II)=5
XI=1,NM
                                                       CALL GGEOT (D
[F2=1
30 5 KI=1,NM
[]=KI
                                                                                                                                                                                                                                                                                                                                                                     16 KI=2, NM
                                                                                                                                          KI = 1 , NM
                       m4
                                                                                            62
                                                                                                                                                                                                                                                                                                                                                            26
                                                                                                                                                                   ~ &
                                                                                                                                                                                                                                                                                                                                              15
```

GO TO 15 22 CALL SIX (1,1J,TIME,1A1,1Q1,QU1,1G1,TW1,IP1,K2,NP,X1,V1,V1,NM,NS) 60 TO 15 23 CALL SEVEN (J,1J,TIME,1A2,1Q2,QU2,1G2,TW2,IP2,K3,NP,X2,V2,Y2,NM,NS ÄLL TWŐ (J.TIME, EVENT, IEVENT, QU2, IG2, IQ2, IP2, IF2, XM2,P2, DSEED2,DS ED8,K3,NP, X2,Y2,V2,NM,NS,T.OH2) O TO 15 19 CALL THREE (L,TIME,EVENT, IEVENT, QU3, IG3, IQ, IP, IF, XM3,P3, DSEED3,DS |
LEED6, DS EED9,A,B,KI,NP,X,Y,V,NM,NS,IA,IR,IS,TOH)
GO TO 15
CALL FOUR (K,IQ,IQ,IP),TIME,EVENT,IEVENT,IM,IG,QS,IA,IA1,IR,IR1,I
LS,IP,A,B,A1,B1,DSEED4,DSEED6,K1,K2,NP,X,Y,V,X1,YI,VI,NM,TI,NS,ISI,
IŢQHL... TO (17,18,19,20,21,22,23,24,25); IL
LONE (1,114, EVENT, IEVENT, QUI, IGI, IPI, IFI, XMI,PI, DSEEDI, DS
D7, K2; NP, XI, YI, VI, NM, NS, TOHI)
TO 15 160 15 21 CALL FIVE (K.12, 102, IP2, TIME, EVENT, IEVENT, IM, IG, QS, IA, IA2, IR, IR2, I 15, IP, A, B, A2, B2, DS EED5, DS EED6, KI, K3, NP, X, Y, V, X2, Y2, V2, NM, T2, NS, IS2, ITOH) IF(EVENT(IN).GT.EVENT(KI)) IN=KI CONTINUE TIME=EVENT(IN) L=IEVENT(IN)

```
W=0.0
W=(TIME-SS)*X1(K2)
W=(TIME-SS)*X1(K2)
ICH=TOHI+WW
ICH=TOHI-TFI
ICH=TQI-TFI
                                                                                    (K2) = 101
(K2) = 1P1
(K2) = TIME
101.67.0)
```

```
| Second | S
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                          GO TO
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                       WW=0.0
WW=(IIYE-SS)*X2(K3)
TUHZ=TUHZ+WW
IF(1Q2.LT.IF2) GO TO
IG2=IQ2-IF2
IP2=IP2-IF2
K3=K3+1
X2(K3)=IQ2
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                      100
```

```
ME, EVENT, IEVENT, QU3, IG3 [Q, IP, IF, XM3, P3, DSEE 1, NP, X, Y, V, NM, NS, IA, IR, IS, TOH) SEED9
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                         VENTÍNS) 19U3(NS), 163(NS), 2(50), X(NP), V(NP), V
2 J=J+1

1 Q2=1Q2-1F2

1 Q2=1F2-1F2

1 Q2(J)=1F2-1Q2

K3=K3+1

K4-1

K5=K1

K6=K1

K6=K1

K1=1,NM

K6=K1

                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                       100
```

```
SUBROUTINE FIVE (K, IQ, IQ2, IP2, IIME, EVENT, IEVENT, IM, IG, QS, IA, IA2, IR

1, IR2, IS, IP, A, B, A2, B2, DSEED5, DSEED6, KI, K3, NP, X, Y, V, X2, Y2, V2, NM, T2, N

1, IS2, TOH)

1, IS2, TOH)

REAL*8 DSEED5, DSEED6

DIMENSION EVENT(NS), IEVENT(NS), IM(NS), IA2(NS), QS(NS), IG(NS), IA(NS)
                                                                                                                                                                                                                                             FORMAT ( 5 100)
FORMAT ( EVENT LIST IS FULL')
EVENT(II) = TIME+TI
RETIPN:
                                                                                                                                                                                                                           :(EVENT(II).EQ.99999.) GO TO
INTINUE
ITE (6.100)
                                                                                                                                                                                                                                                                                                                                                                                         | 102=0
| IF(IP2.GT.IS2) GO TO I
| K3=K3+1
| X2(K3)=1Q2
| V2(K3)=1P2
| Y2(K3)=TIME
| ID2=IR2-IP2
| IP2=IR2-IP2
                                                                                                                                                                                                                                                                                              IEVENT(II)=4
RETURN
DEBUG SUBCHK
END
                                                                                                                                                                                                                                                                          100
```

```
XZ(KZ)=192
VZ(K3)=1PZ
YZ(K3)=TIME
CALL TEN (K,19,1D2,TIME,EVENT,IEVENT,IM,IG,QS,IA,IA2,IR,IS,IP,A,B,
DO 2 KI=1,NM
                                                                                                                                                                                                                                                           SUBROUTINE SIX (I,1J,TIME,1AL,1QL,QUL,1GL,TWL,1PL,KZ,NP,XL,VI,YL,N
M.NS)
DIMENSION IAI(NS),1GL(NS),QUL(NS),XL(NP),VI(NP),Y1(NP)
                                                                                                                                | F(EVENT(II).EQ.99999.) GO TO 3
2 CONTINUE
WRITE (6,100)
100 FORMAT (' EVENT LIST IS FULL')
3 EVENT(II)=TIME+T2
IEVENT(II)=5
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                     IF(IAI(IJ). GE. IGI(N)) GU TO 4
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                        ITINUE
| IA1(1J).LT.ITD) GO TO
| 2 N=1,1
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                            = (TIME-DUI(N)) + I GI(N)
UI(N) = 0.
WI=TWI+W
                                                                                                                                                                                                                                                                                                                                                                                                                                            (1.Eq.0) GO TO
1 N=11
0=1TD+1G1(N)
                                                                                                                                                                                                                                                                                                       2=K2+1

1(K2)=101

1(K2)=101

1(K2)=101

2=K2+1

1(K2)=101

1(K2)=101

1(K2)=101

1(K2)=101
                                                                                                                                                                                                                                 DEBUG SUBCHK
END
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                       1 00 10 8 1.1 WE 0.1 WE 1.1
```

```
ŘETÜRN
DEBUG SUBCHK
END
SUBRDUTINE SEVEN (J,IJ,TIME,IA2,IQ2,QU2,IG2,TW2,IP2,K3,NP,X2,V2,Y2
SUBRDUTINE SEVEN (J,IJ,TIME,IA2,IQ2,QU2,IG2,TW2,IG2,TW2,IP),
I,NM,NS)
DIMËNSION IA2(NS),IG2(NS),QU2(NS),X2(NP),V2(NP),Y2(NP)
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                             iA2(IJ).LT.ITD) GO TO 3
2 N=1, J
                                                                                                                         DO 7 N=1 I

IF(QUI(N).EQ.O.) GO TO 7

MN=MM+1

QUI(MM)=QUI(N)

IGI(MM)=IGI(N)
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                  |=0.
|=(TIME=QU2(N))*1G2(N)
|U2(N)=0.
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                K3=K3+1

X2(K3)=102

V2(K3)=102

V2(K3)=10
62
```

```
SUBROUT INE EIGHT (L,K,IJ,TIME,EVENT,IEVENT,IQ,IA,IAI,IA2,IG,QS,IM, IQU3,IG3,A,B,AI,BI,A2,B2,TW3,TW4,TW5,DSEED4,DSEED5,KI,NP,X,Y,V,NM,N IS,IP)
REAL*B DSEED4,DSEED5
DIMENSION EVENT(NS),IEVENT(NS),IAI(NS),IAZ(NS),IG(NS),QS(NS),IM(NS),QU3(NS),IG3(NS),Z2(50),X(NP),Y(NP),RI(I),RZ(
                 13=0

10=7

1F(QU2(N).EQ.O.) GC TO 7

13=31=1

QU2(JJ)=QU2(N)

1G2(JJ)=1G2(N)
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                8
                                                                                                                                             J=JJ
RETURN
DEBUG SUBCHK
END
5
```

```
10 CONTINUE

10 (AN) = 0

10 (AN) = 0

11 (AN) = 0

11 (AN) = 0

11 (AN) = 0

11 (AN) = 0

12 (AL) = 1

13 (AN) = 0

14 (AN) = 0

15 (EVENT (II) = 6

16 (AN) = 0

17 (AN) = 0

18 (AN) = 0

19 (AN) = 0

10 (AN) = 0

10 (AN) = 0

10 (AN) = 0

11 (AN) = 0

12 (AN) = 0

13 (AN) = 0

14 (AN) = 0

15 (AN) = 0

16 (AN) = 0

17 (AN) = 0

18 (AN) = 0

19 (AN) = 0

10 (AN) = 0

10 (AN) = 0

11 (AN) = 0

12 (AN) = 0

13 (AN) = 0

14 (AN) = 0

15 (AN) = 0

16 (AN) = 0

17 (AN) = 0

18 (AN) = 0

19 (AN) = 0

10 (AN) = 0

10 (AN) = 0

11 (AN) = 0

12 (AN) = 0

13 (AN) = 0

14 (AN) = 0

15 (AN) = 0

16 (AN) = 0

17 (AN) = 0

18 (AN) = 0

19 (AN) = 0

10 (AN) = 0

10 (AN) = 0

10 (AN) = 0

11 (AN) = 0

12 (AN) = 0

13 (AN) = 0

14 (AN) = 0

15 (AN) = 0

16 (AN) = 0

17 (AN) = 0

18 (AN) = 0

18 (AN) = 0

18 (AN) = 0

19 (AN) = 0

10 (AN) = 0
```

```
TE (6,100)
L GGAMS (DSEED5,A2,B2,1,Z2,R2)
NT(II)=TIME+R2(I)
ENT(II)=7
                                                                                                                                                                                                                                                                                         I = KI
IF(EVENT(II).EQ.99999.) GO TO 26
CONTINUE
WRITE (6,100)
IF(IA(IJ).GE.IG(N)) GO TO 19
IG(N)=IG(N)-IA(IJ)
W=(TIME-QS(N))*IA(IJ)
TW5=TW5+W
IAB=IAB+IA(IJ)
GO TO 21
                                                                                                                                                                                                                                                                                                                         6 CALL GGAMS (DSEED5,42,82,1

EVENT(II)=TIME+R2(I)

IEVENT(II)=7

IA2(II)=IA8

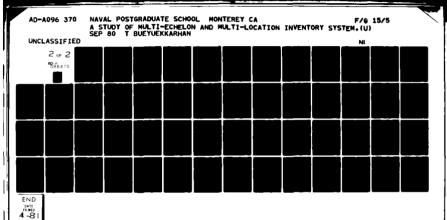
7 NN=0

7 NN=0

1 F(QS(N)-EQ.0.) GO TO 30

NN=NN+1
                                                                                                                                                                                                                                                                   II) = I AA
AB. EQ. 0) GO TO 27
5 KI = 1, NM
                                                       IAA: IAA+1G(N)
IAA(IJ)=IAA(IJ)-IG(N)
IAA(= TWA+W
QS(N)=0.
IG(N)=0.
IAA(N)=0.
IAA(N)=0.
                                                                                                                                                                                                                                                                                                         25
                                                 18
                                                                                                                                                                                                                                                                                                                       26
                                                                                                                                                                                                                                                                                                                                                       27
```

```
| FILLHIND | NE . 1) GO TO 28 | IM (NI) = 29 | GO | NI | SO TO 28 | GO | NI | SO TO 29 | GO | NI | SO TO 20 | GO | NI | GO | THE | GO | TO 20 | THE | THE | GO | TO 20 | THE | T
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                         28
29
                                                                                                                                                                                                                     30
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                       100
```



DTIC

```
I FEEVENTING

3 CONTINUE

4 CALL GGAMS (DSEED61A,B,1,Z,R)

FEVENT(II)=IME+R(I]

I EVENT(II)=IME+R(I]

I EVENT(II)=IME+R(I]

I FEID+IR

I F(IP,LT,IS)

CALL GGAMS (DSEED4,AI,BI,1,ZI,RI)

I F(IN,I)=IME

CONTINUE

I F(EVENT(II)=IME+R(II)

I F(EVENT(II)=IME+R(II)

I F(EVENT(II)=IME+R(II)

I F(EVENT(II)=IME+R(II)

I F(EVENT(II)=IME+R(II)

I F(EVENT(II)=IME+R(II)

I F(EVENT(III)=IME+R(II)

I F(IP,LT,IS)

I F(IP,LT
```

```
72 ML=0

72 ML=0

10 = ML+1

10 = ML+1

10 = ML+1

11 1 = ML+1R

12 ML = ML+1R

13 ML = ML+1R

14 ML = ML+1R

14 ML = ML+1R

15 ML = ML+1R

16 ML = ML+1R

16 ML = ML+1R

17 ML = ML+1R

18 ML = ML+1R

18 ML = ML+1R

18 ML = ML+1R

19 ML = ML+1R

19 ML = ML+1R

19 ML = ML+1R

10 ML = ML+1R

10 ML = ML+1R

10 ML = ML+1R

11 ML = ML+1R

12 ML = ML+1R

12 ML = ML+1R

13 ML = ML+1R

14 ML = ML+1R

15 ML = ML+1R

16 ML = ML+1R

17 ML = ML+1R

18 ML = ML+1R

18 ML = ML+1R

19 ML = ML+1R

10 ML = ML+1R

1
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                        V(KI)=IP

V(KI)=IIME

IN=KA|=IIME

IN=KA|=INE

IN=INE|=INE

IN=INE|=INE

IN=INE|=INE

IN=INE|=INE

V(KI)=INE

                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                       ¥
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                      13
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                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                  12
```

```
IF [10.5 E. 10.2) GO TO 6

CALL GGAMS (DSEED5.A2.82.11.22.R2)

I = KI = I NM

I =
```

```
| F(IP.LI.IS) GD TO 72
| A(II) = ML*IR
| KI=KI+1
| X(KI) = IQ
| V(KI) = IP
| Y(KI) = IP
| Y(KI) = IME
| Y(KI) = IME
| Y(KI) = IME
| Y(KI) = Y(
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APPENDIX D

Simulation Program; No Versatec Plotter Output

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OSEE04, DSEED5, DSEED6, DSEED7, OSEED8, DSE
                                 (900)
I A2(9
IK2(1)
                                 1000
                                        -0-
                                 (900), TAI(900), WZ(50), IKI(
                                 1M(903)
1000
1S(501, W
                                          -O3
                                   ~- ~
                                 (900
(500)
(500)
(500)
                               162(903);
(50) WK (A (10) AK
    DSEED1, DSEED2, DSEED3
                                  (900)
000)
100)
122
(0)
1, KA
                                                                                                                                00000000
                                 VENT (61.90 21.50 21.50 ), $3.00
```

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TURO0790
TURO0800
TURO0810
TURO0830
TURO0830
TURO0850
                                                                                       TURO0890
TURO0900
TURO0920
TURO0930
TURO0930
TURO0950
                                                                                            (OSEED1, XM1, 1, S1)
(OSEED7, 1, P1, WZ, IK1)
                                                                                                  3 KI=1,NM
KI
EVENT(II).EQ.99999.) GO TO
--
                                                             150
                        100
                            110
                                   120
130
                                                     140
```

```
6 KI=2,NM
EVENI(IN).GT.EVENT(KI)) IN=KI
\infty
                                                                                                                                                                                  F(EVENT(II).EQ.99999.) GO TO ONTINUE
                                                                                        F(EVENT(II).EQ.99999.) GO TO ONTINUE
                                                                                                                                                          EVENT(11).EQ.9999.1
                                                                                              UE
[ I)=TIME+S3(I)
[( II)=3
KI=I,NM
                                                                                                                                                                                                         IEVENT( II )= 9
GO TO 26
EVENT( IN) = 99999.
IEVENT( IN) = 0
                                                                                                                                                                                                       I)=YEAR
II)=9
                                                                              KI = I , NM
                                                                                   =K1
                                                                                                                    =KI
                                                                                                                                                                                                                        15
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20 CAL
15 11
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21 CAL
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TUR02340
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                    TI
TIME, EVENT, 1EVENT, QU2, 1G2, 1Q2, 1P2, 1F2, XM2, P2, DSFTI
OH2, $52, 1QQ21
OB
           1111
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                               EVENAL

SSE ENTAL

SSE
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                  5
100
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```
-102
(DSEED2,XM2,1,S2)
(DSEED8,1,P2,WS,IK2)
WW=(TIME-SS2)*192

TOH2=TOH2+WW

IF(IQ2-LT-IF2)

IQ2=IQ2-IF2

IQ2-IF2

IQ2-I
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```
CYCL
      0F
      END
 INE SIX (1,1J, TIME, IA1,1Q1,QU1,1G1,TW1,IP1,NM,NS)
ON IA1(NS),1G1(NS),QU1(NS)
      THE
      AI
      SYSTEM
      0F
     EXPECTED NET INVENTORY
2
```

```
END OF
                      SEVEN (J. IJ, TIME, 1A2, 1Q2, QU2, 1G2, TW2, IP2, NM, NS)
A2(NS), IG2(NS), QU2(NS)
1 1Q2
1 1Q2
1 EXPECTED NET INVENTORY OF SYSTEM 2 AT THE END
END

SUBROUTINE SEVEN (J, IJ, IJ, IIM

DIMENSION IAZ(NS), IGZ(NS), IGZ(NS),
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                       J=JJ
RETURN
DEBUG SUBCHK
END
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                               S
```

```
60
  250
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```
9 U3 (N) = 0.

1 G3 (N) = 0.

1 G3 (N) = 0.

1 F (LU 3 (N) E Q. 0.) GO TO 7

1 L = (LL 1) = QU3 (N)

1 G3 (LL 1) = GU3 (N)

1 G5 (LL 1) = GU3 (N)

1 G5 (LL 1) = GU3 (N)

1 G5 (LL 1) = GU3 (N)

1 G6 (LL 1) = GU3 (N)

1 G7 (LL 1) = GU3 (N)

1 G7 (LL 1) = GU3 (N)

1 G6 (N) = 0

1 G7 (N) = 0

1 G7
```

32

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œ

5

į.

```
00)
(DSEED5,A2,B2,1,Z2,R2)
TIME+R2(1)
IG(N) = 0

IF(IAB.EQ.0) GD TD 31

II=KI INM

II=KI II II II EQ.99999.) GD TD II

CONTINUE

CONTI
```

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     25
      26
          28
           30
    54
       27
```

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TURNO 11100

TURNO 11100
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| FORTING | FORT
                160=100-101

16(1P-31-15) 33 TO 14

10 % KI=1.NM

1 = KI

1 F(EVENT(II).EQ.9999.) GO TO 10

CONTINUE

WRITE (6,100)
                                                                                                                                                                                                                                                                                                                   2
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                         7
                                                                                                                                                                                                                                                                                                               71
```

```
19
                                                             0 ML=ML+1

IP=IP+IR

IF(IP-LT-IS) GO TO 70

IA(II)=ML*IR

DO 60 KT=1,10

LS=KI

IF (AK(LS).EQ.O.0) GO TO 6

0 CONTINUE

1 KA(LS)=IQQ

AK(LS)=EVENT(II)

GO TO 14
                           100
                                                                         60
```

GGAMS (DSEED5,A2,B2,1,22,R2 KI=1,NM F(EVENT(II).EQ.99999.) GO TO ONTINUE RITE (6.100) ARITE CALL GGA:

CALL GGA:

EVENT([1] = 1,

IEVENT([1]) = 8

TI ML=ML+1

IP=IP+IR

IP=IP+IR

IF([P-LT-IS) GD TD 71

IF([P-LT-IS) GD TO 71

IF([P-LT-IS) GD TO 71

IF([N-ML+1]) = ML+1R

IF([N-ML+1]) = ML+1R

IF([N-ML+1]) = IQQ

KA([S) = IQQ 100) |S (DSEED6,A,B,1,Z,R) |=TIME+R(1) |=8 GT.IS) GD TO 14 KI=1,NM CONTINUE AAACLS | 100 CONTINUE CO ø

TURO8650 TURO8650 TURO8670 TURO8690 TURO8700 TURO8720 TURO8730

IEVENT(II)=8
ML=0
'2 ML=ML+1
IP=IP+IR
IF(IP,LT,IS) GO TO 72
IF(IP,LT,IS) GO TO 72
IF(IP,LT,IS) GO TO 72
IF(IP,LT,IS)
IF(IP,IS)

APPENDIX E

Early Warning Simulation Program

```
, DSEED8, DSET
                                          2000
                                          001
A2(
2(1)
      , DSEED4, DSEED5, DSEED6, DSEED7
                                       9000
                                          0-X
                                        900), QUI (900)
900), IAI
12(50), IK
                                       900), IM(902)
163(900), IS(
50), W(50), W(101)
                                     JEVENT (90
22(900) 16
150) WK [50
1(10) AK(1
       DSEED3
       0S EE 01, 0S EE 02,
REAL ***

Define State S
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1UR009930
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1UR009930
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                    TUR00190
TUR00800
TUR00810
TUR00830
TUR00830
TUR00850
TURO0640

TURO05500

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TURO06500

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TURO06500
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                           (DSEED1, XM1, 1, S1)
(DSEED7, 1, P1, WZ, IKI)
                                                                                                                                                                                                                                                                                                                                151, 71
         KAKIN = 0

AKAIN = 0

AKAIN = 0

IF = 0

IF = 0

IF = 0

IF = 0

ID = 10

I
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                             150
                                                                                                                                                                                                                                                                                                                                                                                                                                           120
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                         130
```

IF(EVENT(II).EQ.99999.) GO TO CONTINUE EVENT(II)=TIME+S1(I) IEVENT(II)=1 CALL GGEXN (DSEED2,XM2,1,S2)	ALL GGEOT (DSEEDB;1,P2;WS; F2=1 O 5 KI=1,NM I=KI FFVENT(II),FO,99999,1,GO	CONTINUE EVENT(II) = TIME+S2(I) IEVENT(II) = 2 CALL GGEXN (DSEED3,XM3,1,S3) CALL GGEOT (DSEED9,1,P3,WK,IK3				- HOOM - C	EVENT(1
4		rv o	~ 8	90	177	64	9 9

```
10KG
22 CALL SIX (I,1J,TIME, IAI, IQI, QUI, IGI, TWI, IPI, NM, NS)
10KG
23 CALL SEVEN (J,1J,TIME, IAZ, IQZ, QUZ, IGZ, TWZ, IPZ, NM, NS)
10KG
24 CALL SEVEN (J,1J,TIME, IAZ, IQZ, QUZ, IGZ, TWZ, IPZ, NM, NS)
10KG
25 TAL SEVEN (J,1J,TIME, EVENT, IEVENT, IQ IA, IAI, IAZ, IG, QS, IM, QU3, IGTURG
13, AB, BI, AZ, BZ, TW3, TW4, TW5, DSEED4, DSEED5, NM, NS, IP, KA, AK, I QQI, TURG
13, AB, BI, AZ, BZ, TW3, TW4, TW5, DSEED4, DSEED5, NM, NS, IP, KA, AK, I QQI, TURG
14 TW4, YEAR
14 TW4, YEAR
15 TW4, YEAR
16 TW4, YEAR
17 TW6 TW4, YEAR
17 TW6 TW4, YEAR
18 TW4, 
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                   SUBROUTINE ONE
EDI, DSEED7,NY,
REAL*8 DSEEDI,
DIMENSION EVEN
              1=1
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| International | Internationa
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2 GD TO 3
3 GD TO 4
2 GD TO 5
3 GD TO 6
3 GD TO 6
4 GD TO 70
4 GD TO 70
5 GD TO 70
6 GD TO 70
7 GD TO 7
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1, KATIO) TAKTION

AND END TO THE CONTROL OF DEMAND IN A PERIOD OF 1=*,110)

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AT THE END OF
RETURN

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```

```
END OF
                                               SEVEN (J. IJ. IIME, IA2, IQ2, QU2, IG2, TW2, IP2, NM, NS)
A2(NS), IG2(NS), QU2(NS)
1 IQ2
1 EXPECTED NET INVENTORY OF SYSTEM 2 AT THE END
00 00
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                 (17)-162(N
(N))*162(N
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                        4 IA2(IJ)=IA2(IJ)-

TWZ=TWZ+W

TWZ=TWZ+W

QUZ(N)=0.

IGZ(N)=0.

CONTINUE

JUSO

JUSO
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                              1 62(JJ) = 1 62 (N)
1 62(JJ) = 1 62 (N)
1 0NT INUE
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                      SUBCHK
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                   J=JJ
RETURN
DEBUG S
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                   S
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A(1J)=1A(1J)-1G3(N)
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1,81,1,21,R1)
                   CALL GGAMS (UDSECTI).
EVENT([I])=TIME+RI[I].
IEVENT([I])=A
IAI([I])=IAA
2 IAB=0
DO 13 N=1,K
IF(IM(N).NE.2) GO TO 13
W=0.
W=(TIME-QS(N))*IG(N)
TWS=TWS+W
IAB=IAB+IG(N)
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100) EVENT L MS (DSEED) =TIME+R1

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GGAMS (DSEED4,A1,B1,1,Z1,R1)
(II)=TIME+R1(1)
T(II)=6
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L GGAMS (DSEED5,A2,B2,1,Z2,R2)
NT(II)=TIME+R2(I)
ENT(II)=TA
                IM(N)=0

CONTINUE

IF (IAA.EQ.0) GD TO 24

DO 22 KI=I,NM

I = KI

I = KEVENT(II).EQ.99999.) GO TO 2:

CONTINUE

WRITE ( 6,100)
                                                                                                                                                                                                                                                                                                     UD 30 N=1 K

IF(QS(N).EQ.0.) GO TO 30

NN=NN+1

IF(IM(N).NE.1) GO TO

IM(NN)=1
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28 IM(NN) = 2
29 QS(NN) = QS(N)
IG(NN) = IG(N)
30 CONTINUE
K=NN
31 RETURN
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CALL GGAMS (DSEED4,A1,B1,1,Z1,R1)

I FEEVENT (II) . EQ. 99999.) GD TO 2

CONTINUE

WRITE (6 100)

EVENT(II) = TIME+R1(I)

SEVENT(II) = TIME+R1(I)

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I G(K) = ID - IQ

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GGAMS (DSEED5,A2,B2,l,Z2,R2)
KI=1,NM
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I I = KI

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I F (EVENT (II) - EQ. 99999.) GO TO 2

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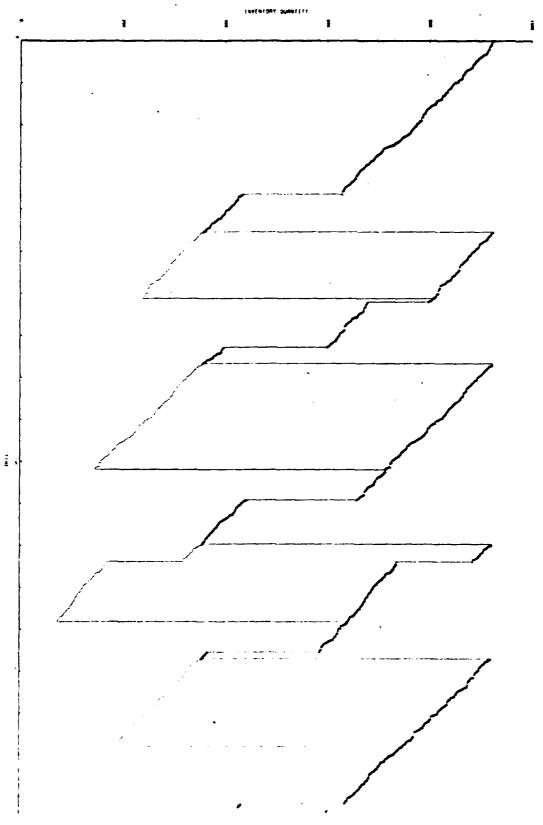
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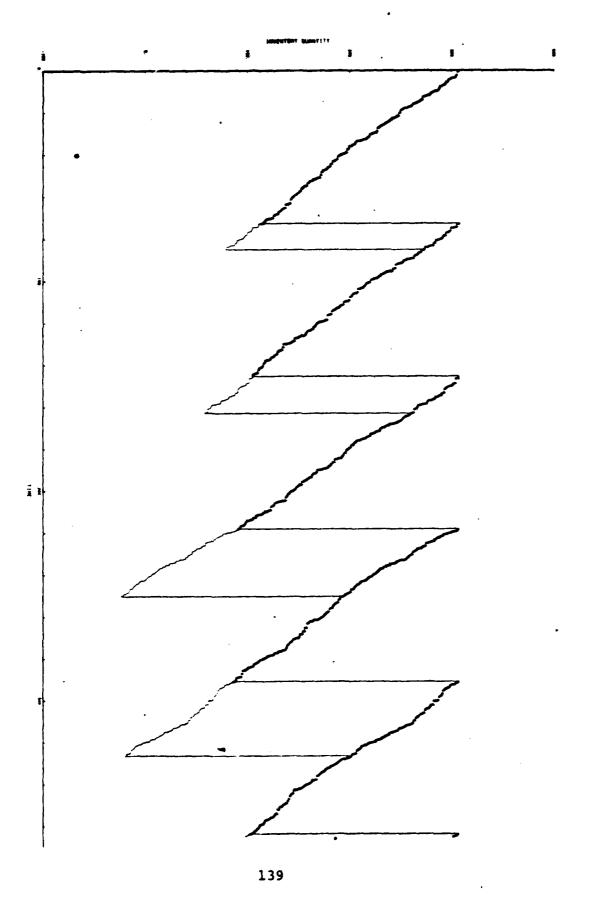
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CONTINUE
KA(L5)=1QQ
AK(L5):EVENT(II)
IP=IR+IS
GO TO 14
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IF(EVENT(II).EQ.99999.)
CONTINUE
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APPENDIX F Versatec Output of the Program in Appendix C





APPENDIX G

TI-59 Calculator Program for Analytical Solutions for Periodic Review Systems

This program was written for probabilistic periodic review inventory models having gamma distributed lead times and Poisson arrivals and uses. Normal distribution for demand during lead time instead of Negative Binomial.

Input Requirements

The following variables should be stored in the registers shown before the variables.

STO 01 = λ (arrival rate per day)

STO 02 = C

STO 03 = I

STO $04 = \Pi$

STO 05 = J

STO 06 = A

STO 07 = α -1

STO 08 = β

This program also requires the TI-59 applied statistics module.

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	В	S
	С	η(r)/period
	D	η(r)/year
	E	Annual review and order cost
	Α'	Annual inventory carrying cost
	В'	Annual shortage cost
	c'	Total annual variable cost

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APPENDIX H

Variable Definitions for Simulation Programs

IQ = Net inventory

IP = Inventory position

IPP = Second inventory position for early warning system

IR = Order quantity

IS = Reorder level

ID = Number of items demanded by systems

IF = Number of items demanded per demand by ships

QU = Ship queue

QS = Lower echelon's group demand queue in the main system

IG = Amount of demand for each demand waiting in the
 queue

IA = Amount of shipment arrived

IM = Index. If it is equal to 1, that means that the demand waiting in the main system queue to be filled belongs to System One.

TW = Total waiting time

TOH = Total average on-hand inventory

T = Length of a period

EVENT = This indicates the subroutines

IEVENT = This indicates time of subroutines scheduled

X = Net inventory variable for Versatec plotter

V = Inventory position variable for Versatec plotter

Y = Time for Versatec plotter

WK = Work space for geometric random variable

WS = Work space for geometric random variable

WZ = Work space for geometric random variable

S = Exponential random number

IK = Geometric random number

WW = Increment

I = Indicates the number of ships waiting in the ship
 queue at System One

J = Indicates the number of ships waiting in the ship
 queue at System Two

L = Indicates the number of ships waiting in the ship
 queue at the main system

K = Indicates the number of demand batches waiting
in the group demand queue at the main system

SS = Time indicator

ML = Multiplier for the number of batches of demand to be ordered from outside supplier

KA = Indicates the last change on net inventory of the
 main system in order to get the number of items
 demanded in a lead time

AK = Time of last change

A = Scale parameter for a gamma distribution

B = Shape parameter for a gamma distribution

XM = Ship arrival rate per day

P = Probability of success for geometric distribution

BIBLIOGRAPHY

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